"Other Side, the final frontier. These are the secret tips for the advanced sorobanist. Its life-long mission, to explore strange new number world, to seek out new tips and new observations, to boldly go where no one has gone before..." just kidding... ;-)

INTRODUCTION: What is the Other Side?

In Soroban, each of the rods represents a single digit of the number depending on the position of the beads. Each of the four earth beads are counted as '1' when they are moved to the reckoning bar, and one heaven bead is counted as '5' when it is moved to the reckoning bar, so you can easily recognize a big cluster of beads attached to the reckoning bar.

These stand-out cluster of beads are sometimes called '表面' by Japanese advanced users. ('表面' ('Omote-men'), literally means 'obverse side', but don't confuse with a Japanese word 'Hyou-men', which is represented by the same Kanji-character... which means 'surface'). And if there is an obverse side, there always be a reverse side. All the beads NOT moved to the reckoning bar side compose '裏面' ('Ura-men', sometimes called 'Ri-men', literally means 'reverse side'), but I don't want you to turn-over or flip the soroban (as a matter of fact, there is such a reversible soroban, by the way), I think 'the Other Side' is a reasonable name.

When you read numbers from the Other Side, you have to add '1' to the least significant non-zero digit. For instance, when the obverse side of the soroban reads '1230', you have to read as '8770'.

Yes, it is the same as the 10's complement of the decimal number in math term, and it could be represented as '10^n - X' (provided that 'n' is the number of the digits for X). Speaking about the value of 'n', by the way, in computer science, accumulators inside CPU have certain amounts of bits such as 32 or 64, and their complement representation is based on '2^32 - X' or '2^64 - x' even if the value X has just a few bits. Similarly, we can think of the complement number for '1230' is '999999999998770' for 15 rods soroban. But for now, let's assume that the 'n' is the number of the digits for X.

IS IT A REPRESENTATION JUST FOR NEGATIVE NUMBERS?

Absolutely NOT. A wise person once said...

"Placing a negative value is a way to enter the world of the Other Side, but the Other Side does not necessarily mean it's the negative world. -- Master it, anyway."

Don't worry, even if you don't understand his word right now, you will know what it means when you read through this document.

So, let's get started with four basic arithmetic operations.

[SUBTRACTION]

Subtracting larger number than minuend is the easiest way to enter the world of the Other Side. When you do this, assume there is '1' at the Boundary Rod (BR).

BR is determined by looking at the subtrahend. For instance, if you were to calculate 123 - 4567, then the number of digits of subtrahend is 4 (which is the 'n' we previously discussed). It means the 5th digit from the least significant digit becomes the BR for the operation. In other words, we have to think of the minuend is 10123 rather than 123. (we call it 'borrow 1 from the digit'.)

```
So, 123 - 4567 = -4444
ABCD  abcde  : place minuend and subtrahend
4567    123  : assume there is '1' at [a] and subtract [ABCD] (4567)
4567   5556  : read the other side (which is 4444, negative)
```

Note: If you're in the world of the Other Side, you always have to be aware where the BR is (in this case, [a]).

--Why borrow '1' from the digit?--

If you don't borrow '1' from BR, you would end up suffering tedious operation. For instance, calculation for 123 - 4567 = -4444 would be like this.

```
ABCD  abcdefghijklmno  : place minuend and subtrahend
4567  000000000000123  : do the subtract operation...
4567  999999999995556  : it's like the computers' representation for
the complement numbers...it isn't wrong after all, but it's a job
for computers, not you.
```

Moreover, if we use an infinite rods soroban, we would end up with an infinite loop of operation. But if we mentally add '1' to the BR (which means add '10000' at first), we can prevent happening such an operation. That's why we borrow '1' from the digit. And if you borrow something, you have to return (but in subtraction/addition case, no interest payment is required. :-))

[ADDITION]

Now, let's exit from the Other Side. The only way to exit from the Other Side is adding the bigger value than the absolute value of summand which is on the board. And all you have to do is ordinary add operation except ignoring carry-over to the BR (ignoring carry-over is the same as returning the borrowed '1').

```
So, -4444 + 4567 = 123
ABCD  abcde : place summand and addend
4567   5556 : The board presents 5556, but Other Side denotes 4444
            : just add [ABCD] (4567), but ignore carry-over to [a]
```

4567   0123 : read the obverse side (which is, of course, 123, positive)

[MULTIPLICATION]

Let's proceed to multiplication. -4444 x 32 = -142208. In this example, I use 留頭頭乗法 (Keeping the initial digit of the multiplicand, and multiply from the initial digit of the multiplier), but any multiplication methods, including Special Operations could be applied.

```
AB  abcdefg  : place multiplicand and multiplier (BR is [a]).
32  05556    : looking at [e] (6), multiply [AB] (32),
             :   and place the result to [efg].
32  0555192  : looking at [d] (5), multiply [AB] (32), clear [d],
             :   and add the result to [def].
32  0551792  : looking at [c] (5), multiply [AB] (32), clear [c],
             :   and add the result to [cde].
32  0517792  : looking at [b] (5), multiply [AB] (32), clear [b],
             :   and add the result to [cde].
32  0177792  : multiplication finished, but the initial value 5556
contains
             :   the value '1' borrowed from [a],
             :   and we have multiplied it by [AB] (32),
             :   it means we must compensate it (with interest
payment!).
             : so, subtract 32 ([AB] x '1') from [bc] (rods adjacent
to BR).
32  0857792  : read the other side (which is 142208, negative)
```

Next example is -12 x 11 = -132

```
AB  abcde  : place multiplicand and multiplier (BR is [a]).
11  088    : looking at [c] (8), multiply [AB] (11),
           :   and place the result to [cde].
11  08088  : looking at [b] (8), multiply [AB] (32), clear [b],
           :   and add the result to [bcd].
11  00968  : subtract 11 ([AB] x '1') from [bc] (rods adjacent to
BR)
11  09868  : read the other side (which is 132, negative)
```

In any cases, you have to make sure where the subtraction performed, to properly return '1' (compensate).

*The Principle

Multiplication could be represented as...

P = A * B (provided that P is product, A is multiplicand, and B is multiplier)

As I mentioned before, Obverse number and the Other Side number have bi-directional relationship represented by '10^n - X', so if the value A is placed on the Other Side, the value of the obverse side should be '10^n - A'. Let's transform the above expresion...

```
    P = (10^n - A) * B
      = 10^n*B - A*B
```

So, if we multiply the soroban's obverse value by B, all we have to do is just subtract '10^n*B' then we can expect to get '-A*B'. (minus sign means the value is on the Other Side)

To cultivate a better understanding, let's see what would be like if we do this calculation without placing BR, I mean, computer like operation using 26 rods soroban.

```
AB  abc...opqrstuvwxyz  : place multiplicand and multiplier.
32  999...9999995556    : looking at [x] (6), multiply [AB] (32),
                        :   and place the result to [xyz].
32  999...999999555192  : looking at [w] (5), multiply [AB] (32),
                        :   clear the [w], and add the result to
[wxy].
32  999...999999551792  : looking at [v] (5), multiply [AB] (32),
                        :   clear [v], and add the result to [vwx].
32  999...999999517792  : looking at [u] (5), multiply [AB] (32),
                        :   clear [u], and add the result to [uvw].
32  999...999999177792  : looking at [t] (9), multiply [AB] (32),
                        :   clear [t], and add the result to [tuv].
32  999...999993057792  : looking at [s] (9), multiply [AB] (32),
                        :   clear [s], and add the result to [stu].
32  999...999931857792  : looking at [r] (9), multiply [AB] (32),
                        :   clear [r], and add the result to [rst].
32  999...999319857792  : looking at [q] (9), multiply [AB] (32),
                        :   clear [q], and add the result to [qrs].
32  999...993199857792  : looking at [p] (9), multiply [AB] (32),
                        :   clear [p], and add the result to [pqr].
32  999...931999857792  : looking at [o] (9), multiply [AB] (32),
                        :   clear [o], and add the result to [opq].
32  999...319999857792  : ... may I give up now? ...   ^_^;
```

If we continue doing this operation beyond the rods of soroban (in

[DIVISION]

Let's proceed to division. -142208 / 32 = -4444. In this example, I use nowadays standard method called '商除法' (Division by using multiplication table), but any division methods, including Special Operations could be applied.

Division is an inverse of multiplication, so we have to add the divisor to the dividend at first, and divide by the divisor. When you add the divisor, match the Most Significant Digit (MSD) of the dividend and the divisor, mentally calculate the sum, and determine whether carry-over would happen. If carry-over won't happen, then imagine there is '9' just left of MSD of the dividend, and match that digit and the MSD of the divisor. In either cases, don't place the carry-overed '1' to the board.

Let's calculate -142208 / 32 = -4444.

```
AB  abcdefg  : place multiplicand and multiplier.
32   857792  : first, match the MSD of dividend ([bc]) and divisor
([AB]), and
             :   determine whether carry-over would happen.
             :   32 + 85 would be 3 digit number (carry-over does
happen!), so add them,
             :   but don't forget to ignore carry-over!
32   177792  : just do the normal divide operation (177792 / 32)
32  5556000  : the answer is 4444, negative.
```

Next example: -132 / 11 = -12.

```
AB  abcde    : place multiplicand and multiplier.
11    868    : first, match the MSD of dividend ([cd]) and divisor
([AB]), and
             :   determine whether carry-over would happen.
             :   This time 86 + 11 would be 2 digit number and
carry-over won't happen,
             :   so imagine there is '9' at [b] and calculate 98 +
11 (=109) and place
             :   '09' to [bc].
11   0968    : just do the normal divide operation (968 / 11)
11  88000    : the answer is 12, negative.
```

*The Principle

Division can be expressed as 'Q = A / B', but it could be rewrite as...

    Q = Q * B / B

Since the dividend 'Q * B' is already placed onto the Other Side, it could be represented as '10^n - Q*B' (provided that n is the number of digits of dividend) on the obverse side. And we don't know the number of digits of Q itself yet, but the answer (Q) would be place onto the Other Side, so we could represent the answer could be represented as '10^m - R' (provided that m is the number of digits of R) on obverse side. Now let's sort out the expression.

```
    10^n - Q*B     : Q could be represented as '10^m - R', so...
  = 10^n - B*(10^m - R)
  = 10^n - 10^m*B + BR
```

If you look at the expression, you could see if we add '10^m*B' and subtract '10^n' (compensate the carry-over), then we would get 'BR'. After that we could divide by 'B', then we would get 'R' itself, which is the obverse side representation of quotient 'Q'.

[NEGATIVE VS. NEGATIVE]

Remember that multiply/divide both negative numbers yields positive result, but you will still in the Other Side after the calculation. (The only way to exit from the Other Side is adding the bigger value than the absolute value of summand which is on the board.)

Let's calculate (14+18-67)x(-24)/(-15)+80 = 24. I think now you can figure out what I do without brief explanations.

```
 abcd
  14  : 14, positive, in obverse side
  32  : 14+18, positive, in obverse side
  65  : 14+18-67, negative, in Other Side
  156 : (14+18-67)x(-24) in the middle of calculation
  916 : (14+18-67)x(-24) completed, positive, in Other Side
   66 : (14+18-67)x(-24)/(-15) in the middle of calculation
   44 : (14+18-67)x(-24)/(-15) completed, negative, in Other Side
   24 : (14+18-67)x(-24)/(-15)_80, positive, in obverse side
```

If you have any questions or find typo/mistake/weird English use, please tell me. :-)

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