Hi all,

Continuing from Yahoo! Groups' MsgNum: 10330

3. '帰一法除法' (Division by Complementary Numbers)

This method is briefly explained in the book by Takashi Kojima, but I would like to explain here in details.

When to apply:
 This technique could be applied when the divisor's initial digit is 8 or 9; or anytime by using 'multiply-same-number-to-both-terms' technique, described in the last post.

Theory and Brief Operation:
 Let's think about the algebraic expression for division below:

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  Q = A / B
  (where Q is quotient, A is dividend, and B is divisor)
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 First, when the initial digit of the divisor is 8 or 9, we can express the value as '10^n - M', for instance 813 can be expressed as 10^3 - 187.
 Let's sort out these expressions.

```
  Q = A / B                           : replace B with (10^n - M)
  Q = A / (10^n - M)                  : multiply both sides by (10^n - M)
  Q * (10^n - M) = A                  : expand the left side
  Q * 10^n - Q * M = A                : add 'Q * M' to both sides
  Q * 10^n = A + Q * M                : divide both sides by '10^n'
  Q = (A + Q * M) / 10^n              : this expression is the gist of the
technique
```

 The last expression tells us how to use this technique.
 Since A is already placed on the board as dividend, all you have to do is, from left rod to right rod, (1) find out an interim quotient intQ in your mind, (2) multiply the intQ by M, and (3) add the result to the board. If you could choose the right interim quotient, that value (interim quotient) should appear on the rod where the right quotient should be placed.

 The merit of this technique comes from that all the operations are done by multiplications and additions (it means easy to determine the interim quotient).

Let's see some examples.

```
Example 1: 641457 / 813 = 789
ABC  abcdef
813  641457 : Set the complement number for divisor 813 (it's 1000 - 813).
187  641457 : Look at the [a], and ponder...
            :   If we treat the value at [a] (6) as interim quotient,
            :   multiplying the value at [ABC] and add the result to [abcd]
            :   makes the value at [a] 7, so the real quotient would be 7.
            :   (It sounds like a daunting task, but actually it's not.
            :    You need to multiply and add against each rod [A], [B],
            :    and [C], but you just need to track the carry-over and
            :    determine whether the value at [a] would change or not,
            :    so you can stop the task whenever if you are confident
            :    that carry-over won't happen any more.)
            : So, multiply 7 by [ABC] (187), and add the result to [abcd].
187  772357 : Look at the [b], and ponder...
            :   If we treat the value at [b] (7) as interim quotient,
            :   multiplying the value at [ABC] and add the result to [bcde]
            :   makes the value at [b] 8, so the real quotient would be 8.
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            : So, multiply 8 by [ABC] (187), and add the result to [bcde].
187  787317 : Look at the [c], and ponder...
            :   If we treat the value at [c] (7) as interim quotient,
            :   multiplying the value at [ABC] and add the result to [cdef]
            :   makes the value at [c] 8, so the real quotient would be 8.
            : So, multiply 8 by [ABC] (187), and add the result to [cdef].
187  788813 : Look at the rest of the value at [def],
            : It's the same value as the divisor (813), so we can safely
            :   increment the last quotient at [c].
            :   (If you accustomed this technique, you can directly
determine
                the real quotient 9 at [c] from previous step [787317].)
187  789000 : done.

Example 2: 7412292 / 927 = 7996
ABC  abcdefg
927  7412292 : Set the complement number for divisor 927 (it's 1000 - 927).
073  7412292 : Look at the [a], and ponder...
             :   If we treat the value at [a] (7) as interim quotient,
             :   multiplying the value at [ABC] and add the result to
[abcd]
             :   (don't forget the existence of digit [A], so addition must
be
             :    performed on [abcd]!)
             :   makes the value at [a] 7, so the real quotient would be 7.
             : So, multiply 7 by [ABC] (073), and add the result to [abcd].
073  7923292 : Look at the [b], and ponder...
             :   If we treat the value at [b] (9) as interim quotient,
             :   multiplying the value at [ABC] and add the result to
[bcde]
             :   doesn't affect the value [b], so the real quotient would
be 9.
             : Multiply 9 by [ABC] (073), and add the result to [bcde].
073  7988992 : Look at the [c], and ponder...
             :   If we treat the value at [c] (8) as interim quotient,
             :   multiplying the value at [ABC] and add the result to
[cdef]
             :   affect the value [c], so the real quotient would be 9.
             : Multiply 9 by [ABC] (073), and add the result to [cdef].
073  7995562 : Look at the [d], and ponder...
             :   If we treat the value at [d] (5) as interim quotient,
             :   multiplying the value at [ABC] and add the result to
[defg]
             :   doesn't affect the value [d], so the real quotient would
be 5.
             : Multiply 5 by [ABC] (073), and add the result to [defg].
073  7995927 : Look at the rest of the value at [efg],
             :   It's the same value as the divisor (987), so we can safely
             :   increment the last quotient at [d].
073  7996000 : Done.


This is the '帰一法除法'.
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The next technique is called '過大商帰一除法'.

4. '過大商帰一除法' (Division by Complementary Numbers with Excessive Quotient)

When we accidentally (or intentionally by advanced user) assume excessive interim quotient (which means the interim quotient is real quotient + 1), sometimes sequences of 9s are appeard on the board.
In that case, we can short cut the operations.

Let's see tha last example: 7412292 / 927 = 7996

```
ABC  abcdefg
927  7412292 : Set the complement number for divisor 927 (it's 1000 - 927).
073  7412292 : Look at the [a], and ponder...
             :   In the previous example, we assume the interim quotient as 7,
             :   but this time, let's assume it as 8.
             : Multiply 8 by [ABC], and add the result to [abcd]
073  7996292 : We assumed the interim quotient as 8, but rod [a] doesn't become 8.
             : That means the value was too large, so we have to rollback the process.
             : But we can think the value [abc] (799) is the right quotients and
             :   continue the operation from here.
             : Look at the rod [d] (6) and yield the complementary number, which is 4,
             :   so multiply 4 by [ABC], and subtract the result from [efg]
073  7996000 : Done.
```

Let's see what we actually did in this example.

Division expression could be represented like this:

$$Q = A / B$$

This could be rewritten as:

$$Q = (Q * B) / B$$

and this could be a hindsight, but we can say we assumed the wrong interim quotient 800 when the actual interim quotient was 799.
That means 800 was the 'excessive quotient'.
So let's rewrite the expression with replacing the quotient and the divisor:

$$Q = ((E - D) * (10^n - M)) / (10^n - M)$$

(By the way, E is 800 and D is 1, in our previous example.)

and it could be rewritten as:

$$Q = (10^n*E - 10^n*D - E*M + D*M) / (10^n - M)$$

The value $(10^n*E - 10^n*D - E*M + D*M)$ is on the board at the first place, (remember, it's the same as (Q * B), or A) and we add the result of 800 * 73 at the first operation, which is E*M, all we have to do next is subtract D*M, then the quotient Q would appear on the board.

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