Now divisions... :-)

1. '省一法除法' (The Elimination of the Initial Digit of a Divisor Beginning with One)

This method is briefly explained in the book by Takashi Kojima, but I would like to explain here in details.

When to apply:
 This technique could be applied when the divisor's initial digit is 1 and the 2nd digit (from the top) is lesser value, such as 0, 1, 2. (The smaller the 2nd digit, the easier to determine the interim quotient.)

#Special note:
While this technique is categorized as 'Special Operations,' which are applicable only in certain (limited) situations, we can apply any divisions with a little bit of twist. Think about the expression A / B = (A * n) / (B * n) becomes always true. So, when we have divisor 643, for instance, we can multiply both divisor and dividend by 2 to make divisor 1286. Furthermore, in this case, multiplying by 16 (to make divisor 10288) is more preferable, because it makes estimation for interimm quotient easier. (Multiplication by 16 is a little bit complicated than multiplying by 2, though.) You can multiply dividend and divisor by 1556, but it might be overkill. :)

Theory and Brief Operation:
 Let's think about the algebraic expression for division below:

```
  Q = A / B
  (where Q is quotient, A is dividend, and B is divisor)
```

 First, when the initial digit of the divisor is 1, we can express the value as '10^n + R', for instance 12345 can be expressed as 10^5 + 2345.
 Let's sort out these expressions.

```
  Q = A / B                                : replace B with (10^n + R)
  Q = A / (10^n + R)                       : multiply both sides by (10^n + R)
  Q * (10^n + R) = A                       : expand the left side
  Q * 10^n + Q * R = A                     : subtract 'Q * R' from both sides
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  Q * 10^n = A - Q * R               : divide both sides by
'10^n'
  Q = (A - Q * R) / 10^n             : this expression is
the gist of the technique
```

The last expression tells us how to use this technique. Since A is already placed on the board as dividend, all you have to do is, from left rod to right rod, (1) find out an interim quotient intQ in your mind, (2) multiply the intQ by R, and (3) subtract the result from the board. If you could choose the right interim quotient, that value (interim quotient) should appear on the rod where the right quotient should be placed.

The merit of this technique mostly comes from reducing initial digit (and following zeroed digits, if any) to operate, but since the divisor's initial digit is 1, interim quotient tends to fall into the exact value or a little bit less value already on the board, which means lesser hustle would be needed.

Let's see some examples...

```
Example 1: 35190 / 102 = 345
ABC  abcde
102  35190  : Clear the initial digit [A] of the divisor.
 02  35190  : Look at the [a], and ponder...
            :   if we treat the value at [a] (3) as interim
quotient,
            :   multiplying the value by [BC] (02) yieds
6...
            :   subtracting 6 from [abc] doesn't affect the
value at [a]...
            :   which means interim quotient 3 is indeed
the real quotient.
            : Subtract 6 from [abc].
 02  34590  : Look at the [b], and ponder...
            :   if we treat the value at [b] (4) as interim
quotient,
            :   multiplying the value by [BC] (02) yieds
8...
            :   subtracting 8 from [bcd] doesn't affect the
value at [b]...
            :   which means interim quotient 4 is indeed
the real quotient.
            : Subtract 8 from [bcd].
 02  34510  : Look at the [c], and ponder...
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```
            :   if we treat the value at [c] (5) as interim
quotient,
            :   multiplying the value by [BC] (02) yieds
10...
            :   subtracting 10 from [cde] doesn't affect
the value at [b]...
            :   which means interim quotient 5 is indeed
the real quotient.
            : Subtract 10 from [cde].
 02  34500  : Done.


This example is too convenient.  Maybe I should regret...
:)
Okay, let's move on to more complicated examples.

Example 2: 4404 / 12 = 367
AB  abcd
12  4404   : Clear the initial digit [A] of the divisor.
 2  4404   : Look at the [a], and ponder...
           :   if we treat the value at [a] (4) as interim
quotient,
           :   multiply the value by [B] (2) yields 8...
           :   subtracting 8 from [ab] makes value at [a] 3
               ...it's smaller than the interim quotient we
assumed.
           :   How about 3 as interim quotient?
           :   multiply the value by [B] (2) yields 6...
           :   subtracting 6 from [ab] makes value at [a] 3
               ...Good!  3 is the real quotient.  Subtract
6 from [ab].
 2  3804   : Look at the [b], and ponder...
           :   if we treat the value at [b] (8) as interim
quotient,
           :   multiply the value by [B] (2) yields 16...
           :   subtracting 16 from [bc] makes value at [b]
6
               ...it's smaller than the interim quotient we
assumed.
           :   if we treat 7 as interim quotient,
           :   the value at [b] changes to 6, smaller
again!
               ...Bad luck.
           :   How about 6?
           :   multiply the value by [B] (2) yields 12...
           :   subtracting 12 from [bc] makes value at [a]
6
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```
               ...Good!  6 is the real quotient.  Subtract
12 from [bc].
 2  3684   : Look at the [c], and ponder...
           :   if we treat the value at [c] (8) as interim
quotient,
           :   multiply the value by [B] (2) yields 16...
           :   subtracting 16 from [bc] makes value at [c]
7
               ...it's smaller than the interim quotient we
assumed.
           :   How about 7 as interim quotient?
           :   multiply the value by [B] (2) yields 14...
           :   subtracting 14 from [cd] makes value at [c]
7
               ...Good!  7 is the real quotient.  Subtract
14 from [cd].
 2  3670   : Done!
```

Are you following?
Then look at the next example. It has a teeny tiny pitfall.

```
Example 3: 9468 / 12 = 789
AB  abcd
12  9468   : Clear the initial digit [A] of the divisor.
 2  9468   : Look at the [a], and ponder...
           :  if we treat the value at [a] (9) as interim
quotient,
           :  multiply the value by [B] (2) yields 18...
           :  subtracting 18 from [ab] makes value at [a] 7
              ...it's smaller than the interim quotient we
assumed.
           :  How about 8 as interim quotient?
           :  multiply the value by [B] (2) yields 16...
           :  subtracting 16 from [ab] makes value at [a] 7
              ...it's smaller than the interim quotient we
assumed.
           :  How about 7 as interim quotient?
           :  multiply the value by [B] (2) yields 14...
           :  subtracting 14 from [ab] makes value at [a]
8!!! ?????
              ...We tried 9 and 8, and we got smaller value
7,
                 but when we tried 7, we got 8.  How could
we get 7 at [a]?
```

```
              As a matter of fact, the right quotient is
indeed 7,
                and we should treat the excess value is an
overflow from [b].
                (it means the rod [b] has a value 10,
instead of 0).
              So, subtract 14 from [ab].
 2  8068   : Resume processing...
           : Since the rod [b]'s value is 10...
           :  if we treat 9 as the interim quotient,
           :  multiply the value by [B] (2) yields 18...
           :  subtracting 18 from [abc] makes value at
[abc] 788.
              ...[a] becomes 7, good... but [b] is 8,
smaller than 9, means no good.
           :  How about 8 as the interim quotient?
           :  multiply the value by [B] (2) yields 16...
           :  subtracting 16 from [abc] makes value at [ab]
79!!!
              ...[a] becomes 7, good... but [b] is 9, not
8...but excess again!!!
              ...same situation is happening here,
              ...so subtract 16 from [abc], treat [b] is 8,
and [c] is 10.
 2  7908   : Resume processing...
           : Since the rod [c]'s value is 10...
           :  if we treat 9 as the interim quotient,
           :  multiply the value by [B] (2) yields 18...
           :  subtracting 18 from [bcd] makes value at
[bcd] 890.
              ...so subtract 18 from [bcd].
 2  78900  : Done!
```

The key points are...
* If the result of the subtraction is smaller than interim quotient, then the interim quotient is too big.
* If the result of the subraction becomes interim quotient + ONE, you're on a right path, but has an overflow.

This is the '省一法除法'.
If you mastered this technique as foundation, you are good to go with the next technique called '過大商省一除法'.

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2. '過大商省一除法' (The Elimination of the Initial Digit of a Divisor Beginning with One, with Excessive Quotient)

When the divisor has more than 3 digits, we sometimes happen to assume the wrong interim quotient which is just 1 larger than the real quotient, and subtract the product from the board.
To recover from this situation is, of course, to add the same value we subtracted and retry with the new interim quotient, or just add the modified divisor, but sometimes we can proceed the calculation from that state, assuming some quotients are already on the board.

```
Example 4: 773742 / 129 = 5999
ABC  abcdef
129  773742  : Clear the initial digit [A] of the divisor.
 29  773742  : Assume interim quotient as 6,
             :   subtract 12 (which is 6 * [B]) from [ab].
 29  653742  :   And subtract 54 (which is 6 * [C]) from
[bc].
 29  599742  : The value of [a] became less than the
interim quotient...
             :   which means we assumed wrong interim
quotient.
             : In other words, we should have subtracted (5
* 29) rather than
             :   (6 * 29).
             : But, in the cases like this, we can assume
some of the proper
             :   quotient are already emerge on the board,
which are 599
             :   (on the rods [abc]).
             : Anyway, we have subtracted larger value (6 *
29) than we should
             :   subtract (5 * 29), that means we are
staring at a good old
             :   'OTHER SIDE' [welcome back to the upside-
down world!! :-) ]
             : If you look at the other side of [def], you
can see the value 258,
             :   which is the doubled value of the original
divisor 129.
             : So, we are going to multiply 2 (because the
value is doubled)
             :   by [B] (2), and add the result to [de].
 29  599782  : And multiply 2 by [C] (9) and, add the
result to [ef].
```

```
 29  599900  : Done.
```

Let's see what we actually did in this example.

Division expression could be represented like this:

$$Q = A / B$$

This could be rewritten as:

$$Q = (Q * B) / B$$

and this could be a hindsight, but we can say we assumed the wrong interim
quotient 600 when the actual interim quotient was 599.
That means 600 was the 'excessive quotient'.
So let's rewrite the expression with replacing the quotient and the divisor:

$$Q = ((E - D) * (10^n + M)) / (10^n + M)$$

(By the way, E is 600 and D is 1, in our previous example.)

and it could be rewritten as:

$$Q = (10^n*E - 10^n*D + E*M - D*M) / (10^n + M)$$

The value (10^n*E - 10^n*D + E*M - D*M) is on the board at the first place,
(remember, it's the same as (Q * B), or A) and we subtract the result of
600 * 29 at the first operation, which is E*M, all we have to do next is
ADD D*M, then the quotient Q would appear on the board.

Let's see another example:

```
Example 5: 600753 / 129 = 4657
ABC  abcdef
129  600753  : Clear the initial digit [A] of the divisor.
 29  600753  : Assume the interim quotient as 5,
             :   multiply [BC] by 5, and subtract the
result from [abc].
 29  455753  : 5 was excessive, and the other side says
[44247].
             : we should add 44247 / 129 = 343 as D, so we
have to
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```
              : multiply 3 by 29, and add the result to
[bc],
              : multiply 4 by 29, and add the result to
[cd], and
              : multiply 3 by 29, and add the result to
[de].
              : But how do we determine the value 343 when
we can't mentally
              :   calculate the value?
              : The easiest way is, for instance,
              : (1) look at the [b] (5),
              : (2) find the complement of the number (which
is 5),
              : (3) multiply [BC] and find out the value at
[b] changes (to 7),
              : (4) find the complement of the changed value
at [b]
              : this process yields 3,
              : so multiply 3 by [B] and add the result to
[bc].
              : multiply 3 by [C] and add the result to
[cd].
 29  464453  : The other side is [5547],
              : look at [c] (4), complement is 6, so
multiply 6 and determine
              : how value changed at [c], (it must be 6),
              : so multiply 4 by [B] and add the result to
[cd].
              : multiply 4 by [C] and add the result to
[de].
 29  465613  : The other side is [387], apply same logic
and yield 3,
              : so multiply 3 by [B] and add the result to
[de].
              : multiply 3 by [C] and add the result to
[ef].
 29  465700  : done.
```

This is the brief explanation of this technique.
If there are any questions, please feel free to ask me.
(I'm studying the technique right now, so I'm not sure I
could answer all the questions, but I'll do my best. :-)

masaaki