

A Japanese book ('新訂珠算教授ノ實際', Educating Soroban in Practice, New Edition, by Gen'ichi Nikai) published in 1939 categorized various ways of multiplication and assessed each of them.

They are (I translated the literal meanings of the name in parenthesis):

1. 留頭尾乘法 (Keeping the initial digit of the multiplicand, and multiply from the last digit of the multiplier)
2. 留頭頭乘法 (Keeping the initial digit of the multiplicand, and multiply from the initial digit of the multiplier)
3. 破頭尾乘法 (Breaking the initial digit of the multiplicand, and multiply from the last digit of the multiplier)
4. 破頭頭乘法 (Breaking the initial digit of the multiplicand, and multiply from the initial digit of the multiplier)
5. 新頭乘法 (New Modified method for Keeping the initial digit of the multiplicand, and multiply from the initial digit of the multiplier)
6. 減一法乘法 (Multiplication with decremented multiplier)

Note: multiplier is always place on the left of the multiplicand.

1. 留頭尾乘法

Calculation sequence for $23 * 45$

```
AB  abcd
45  23   : Multiply 3 by 5 and add the answer to [cd]
45  2315 : Multiply 3 by 4, clear [b], and add the answer to [bc]
45  2135 : Multiply 2 by 5 and add the answer to [bc]
45  2235 : Multiply 2 by 4, clear [a], and add the answer to [ab]
45  1035 : done.
```

Pro:

It's very easy to understand since the calculation sequence is the same as the one in calculating on a paper.

Con:

It's hard to determine where to add the sub-product, when the digits of the multiplier is big. And sometimes, we have to memorize and process temporary carry over. Think about $478 * 96 = 45888$, for instance. (I think this issue doesn't affect 2:5 abacus.)

Note: It is the oldest method and is documented in the Chinese book in Ming dynasty (1368-1644).

2. 留頭頭乘法

Calculation sequence for $23 * 45$

```
AB  abcd
45  23   : Multiply 3 by 4, clear [b], and add the answer to [bc]
45  212   : Multiply 3 by 5, and add the answer to [cd]
45  2135  : Multiply 2 by 4, clear [a], and add the answer to [ab]
45  0935  : Multiply 2 by 5, and add the answer to [bc]
45  1035  : done.
```

Pro:

It's easy to determine where to add the sub-product, and there is no need to memorize the carry over.

It is symmetric with Kijo-ho (the method which employs Chinese division table).

Con:

You have to memorize the digit of the multiplier you are focusing on.

Note: This method was popular, before Japanese curriculum abandoned Kijo-ho.

3. 破頭尾乘法

Calculation sequence for $23 * 45$

```
AB abcd
45 23 : Multiply 2 by 5, clear [c], and add the answer to [bc]
45 103 : Multiply 2 by 4, and add the answer to [ab]
45 903 : Multiply 3 by 5, clear [d], and add the answer to [bc]
45 918 : Multiply 3 by 4, and add the answer to [ab]
45 1035 : done.
```

Pro:

The last digit of the multiplicand is match to the last digit of the result.

Con:

You have to memorize the digit of the multiplier you are focusing on.

Many people feel weird with this sequence.

There is no symmetric division alternatives.

4. 破頭頭乘法

Calculation sequence for $23 * 45$

```
AB abcd
45 23 : Multiply 2 by 4, and add the answer to [ab]
45 0823 : Multiply 2 by 5, clear [c], and add the answer to [bc]
45 0903 : Multiply 3 by 4, and add the answer to [bc]
45 1023 : Multiply 3 by 5, clear [d], and add the answer to [cd]
45 1035 : done.
```

Pro:

You don't have to memorize the digit of the multiplier you are focusing on.

Con:

It's hard to determine where to add the sub-product, when the digits of the multiplier is big.

Note: This method is very popular in competition venue, since these competitors don't place multiplicand and multiplier to the board at all, which is called '両落とし' (literally means 'dropping both terms technique').

In practice, it would be like this:

Calculation sequence for $23 * 45$

```
AB abcd
45 : Multiply 2 by 4, and add the answer to [ab]
45 08 : Multiply 2 by 5, and add the answer to [bc]
45 090 : Multiply 3 by 4, and add the answer to [bc]
45 102 : Multiply 3 by 5, and add the answer to [cd]
45 1035 : done.
```

5. 新頭乘法

Calculation sequence for $23 * 45$

```
AB abcde
45 23 : Multiply 3 by 4, clear [b], and add the answer to [cd]
45 2012 : Multiply 3 by 5, and add the answer to [de]
45 20135 : Multiply 2 by 4, clear [a], and add the answer to [bc]
45 00935 : Multiply 2 by 5, and add the answer to [cd]
45 01035 : done.
```

Pro:

It's easy to determine where to add the sub-product, and there is no need to memorize the carry over.

It is symmetric with current division method.

Con:

You have to memorize the digit of the multiplier you are focusing on.

Note: This method is the most major method in nowadays in Japan.

6. 減一法乘法

Calculation sequence for $23 * 45$

```
AB  abcd
45   23  : Decrement the multiplier
44   23  : Multiply 2 by 4 ([A]), and add the answer to [ab]
44  0823 : Multiply 2 by 4 ([B]), and add the answer to [bc]
44  0903 : Multiply 3 by 4 ([A]), and add the answer to [bc]
44  1023 : Multiply 3 by 4 ([B]), and add the answer to [cd]
44  1035 : done.
```

Pro:

The last digit of the multiplicand is match to the last digit of the result.
You don't have to memorize the digit of the multiplier you are focusing on.

Con:

You have to memorize the digit of the multiplier you are focusing on.
There is no symmetric division alternatives.

Note: The method is categorized as a special operation and isn't teach in schools.

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