Converting Octal to Hexadecimal and Hexadecimal to Octal

Converting Hexadecimal to Octal

This method will work on either soroban or suan pan. Because calculations will be done in octal, I recommend using the abacus in octal mode; that is, let the heaven bead count four and only use three earth beads – this way bead manipulation will be the same as in decimal arithmetic and you only need to remember to use eight's complements instead of ten's complements. Otherwise you will need to use an octal addition table to perform the computations.

The method used is:

1. Clear several rods to zero to be used as an accumulator for the conversion, and select a units rod
2. Add to the units rod, using octal arithmetic, the most significant digit of the hexadecimal number
3. Multiply each digit of the accumulator by 16 (20 in octal), working from left to right, by adding the octal double of the digit to its left neighbor rod, and clear the original rod to zero
4. Add to the units rod, using octal arithmetic, the next most significant digit of the hexadecimal number
5. If the number added in step 4 was not the last digit of the hexadecimal number, go to step 3
6. Else stop

Example: hexadecimal 29C → octal 1234

0000 clear the accumulator (the rightmost rod is selected as the units rod)

0002 add the most significant digit of the hexadecimal number to the units rod

0040 multiply 2 by 16 (octal 20) - double 2 and add it to the left neighbor rod, then clear the 2

0051 add the next hexadecimal digit, 9 (11 in octal), to the units rod (and the rod to its left)

1201 multiply 5 by octal 20 - double 5 (10 decimal = 12 octal) and add it to the left neighbor rod, then clear the 5

1220 multiply 1 by octal 20 - double 1 = 2 and add it to the left neighbor rod, then clear the 1

1234 add the last hexadecimal digit, C (= 14 in octal), to the units rod

stop

Another example: hexadecimal B7D6 → octal 133726

000000 clear the accumulator (the rightmost rod is selected as the units rod)

000013 add (octal) the most significant digit of the hexadecimal number to the units rod

000203 multiply 1 by octal 20 - double 1 and add it to the left neighbor rod, then clear the 1

000260 multiply 3 by octal 20 - double 3 and add it to the left neighbor rod, then clear the 3

000267 add the next hexadecimal digit, 7, to the units rod

004067 multiply 2 by octal 20 - double 2 and add it to the left neighbor rod, then clear the 2

005407 multiply 6 by octal 20 - double 6 (= octal 14) and add it to the left neighbor rod, then clear the 6

005560 multiply 7 by octal 20 - double 7 (= octal 16) and add it to the left neighbor rod, then clear the 7

005575 add the next hexadecimal digit, D (= 15 octal), to the units rod

120575 multiply 5 by octal 20 - double 5 and add it to the left neighbor rod, then clear the 5

132075 multiply 5 by octal 20 - double 5 and add it to the left neighbor rod, then clear the 5

133605 multiply 7 by octal 20 - double 7 and add it to the left neighbor rod, then clear the 7

133720 multiply 5 by octal 20 - double 5 and add it to the left neighbor rod, then clear the 5

133726 add the last hexadecimal digit, 6, to the units rod

stop

Converting Octal to Hexadecimal

This method is best used on the suan pan where numbers up to 15 (hexadecimal F) can be placed on a rod. If you are using a soroban you will need to use some means of placing numbers greater than nine on a rod; for examle, by using a coin or other small obect placed just above the rod to represent a count of 10 added to the number shown on that rod's beads. All calculations will be done in hexadecimal, so a hexadecimal addition table may be used, or alternatively mentally perform the addition in decimal and then convert to hexadecimal.

The method used is:

1. Clear several rods to zero to be used as an accumulator for the conversion, and select a units rod
2. Add to the units rod, using octal arithmetic, the most significant digit of the hexadecimal number
3. Multiply each digit of the accumulator by 8, working from left to right, by adding half of the digit (if the digit is even) or half of the digit minus one (if the digit is odd) to the left neighbor rod. If the original digit was even the original digit is then cleared. If the original digit was odd replace it with eight
4. Add to the units rod, using hexadecimal arithmetic, the next most significant digit of the octal number
5. If the number added in step 4 was not the last digit of the octal number, go to step 3
6. Else stop

Example: octal 1234 → hexadecimal 29C

0000 clear the accumulator (the rightmost rod is selected as the units rod)

0001 add the most significant digit of the octal number to the units rod

0008 multiply 1 by 8 – 1 is odd, so add (1-1)/2 = 0 to the left neighbor rod, then replace the 1 by 8

000A add the next octal digit, 2, to the units rod

0050 multiply A by 8 – A is even so add A/2 = 5 to the left neighbor rod, then clear the A

0053 add the next octal digit, 3, to the units rod

0283 multiply 5 by 8 – 5 is odd, so add (5-1)/2 = 2 to the left neighbor rod, then replace the 5 by 8

0298 multiply 3 by 8 – 3 is odd, so add (3-1)/2 = 1 to the left neighbor rod, then replace the 3 by 8

029C add the last octal digit, 4, to the units rod

stop

Another example: octal 133726 → hexadecimal B7D6

00000 clear the accumulator (the rightmost rod is selected as the units rod)

00001 add the most significant digit of the octal number to the units rod

00008 multiply 1 by 8 – 1 is odd, so add (1-1)/2 = 0 to the left neighbor rod, then replace the 1 by 8

0000B add the next octal digit, 3, to the units rod

00058 multiply B by 8 – B is odd, so add (B-1)/2 = 5 to the left neighbor rod, then replace the B by 8

0005B add the next octal digit, 3, to the units rod

0028B multiply 5 by 8 – 5 is odd, so add (5-1)/2 = 2 to the left neighbor rod, then replace the 5 by 8

002D8 multiply B by 8 – B is odd, so add (B-1)/2 = 5 to the left neighbor rod, then replace the B by 8

002DF add the next octal digit, 7, to the units rod

010DF multiply 2 by 8 – 2 is even, so add 2/2 = 1 to the left neighbor rod, then clear the 2

0168F multiply D by 8 – D is odd, so add (D-1)/2 = 6 to the left neighbor rod, then replace D by 8

016F8 multiply F by 8 – F is odd, so add (F-1)/2 = 7 to the left neighbor rod, then replace the F by 8

016FA add the next octal digit, 2, to the units rod

086FA multiply 1 by 8 – 1 is odd, so add (1-1)/2 = 0 to the left neighbor rod, then replace the 1 by 8

0B0FA multiply 6 by 8 – 6 is even, so add 6/2 = 3 to the left neighbor rod, then clear the 6

0B78A multiply F by 8 – F is odd, so add (F-1)/2 = 7 to the left neighbor rod, then replace the F by 8

0B7D0 multiply A by 8 – A is even, so add A/2 = 5 to the left neighbor rod, then clear the A

0B7D6 add the last octal digit, 6, to the units rod

stop

Converting a Hexadecimal Fraction to an Octal Fraction

This method uses hexadecimal arithmetic.

1. Select a units rod and set the hexadecimal fraction onto the rods to the right of the unit rod
2. Multiply each digit of the hexadecimal fraction by 8, working from left to right, by adding half of the digit (if the digit is even) or half of the digit minus one (if the digit is odd) to the left neighbor rod. If the original digit was even the original digit is then cleared. If the original digit was odd replace it with eight
3. Take the overflow which has spilled over onto the units rod as the next most significant digit of the octal fraction, then clear the units rod
4. If there are remaining non-zero digits in the hexadecimal fraction, go to step 2
5. Else stop

Example: hexadecimal 0.B7D6 → octal 0.557530

0.B7D6 set the hexadecimal fraction onto the rods to the right of the units rod

5.87D6 multiply B by 8 – B is odd so add (B-1)/2 = 5 to the left neighbor rod and replace B with 8

5.B8D6 multiply 7 by 8 - 7 is odd so add (7-1)/2 = 3 to the left neighbor rod and replace 7 with 8

5.BE86 multiply D by 8 - D is odd so add (D-1)/2 = 6 to the left neighbor rod and replace D with 8

5.BEB0 multiply 6 by 8 – 6 is even, so add 6/2 = 3 to the left neighbor rod and clear 6 to 0

0.BEB0 the overflow, 5, is the first digit of the octal fraction (0.50000 so far) – clear the overflow

5.8EB0 multiply B by 8 - B is odd so add (B-1)/2 = 5 to the left neighbor rod and replace B with 8

5.F0B0 multiply E by 8 – E is even, so add E/2 = 7 to the left neighbor rod and clear E to 0

5.F580 multiply B by 8 - B is odd so add (B-1)/2 = 5 to the left neighbor rod and replace B with 8

0.F580 the overflow, 5, is the next digit of the octal fraction (0.55000 so far) – clear the overflow

7.8580 multiply F by 8 - F is odd so add (F-1)/2 = 7 to the left neighbor rod and replace F with 8

7.A880 multiply 5 by 8 - 5 is odd so add (5-1)/2 = 2 to the left neighbor rod and replace 5 with 8

7.AC00 multiply 8 by 8 – 8 is even, so add 8/2 = 4 to the left neighbor rod and clear 8 to 0

0.AC00 the overflow, 7, is the next digit of the octal fraction (0.55700 so far) – clear the overflow

5.0C00 multiply A by 8 – A is even, so add A/2 = 5 to the left neighbor rod and clear A to 0

5.6000 multiply C by 8 – C is even, so add C/2 = 6 to the left neighbor rod and clear C to 0

0.6000 the overflow, 5, is the next digit of the octal fraction (0.55750 so far) – clear the overflow

3.0000 multiply 6 by 8 – 6 is even, so add 6/2 = 3 to the left neighbor rod and clear 6 to 0

0.0000 the overflow, 3, is the next digit of the octal fraction (0.55753 so far) – clear the overflow

stop – all hexadecimal digits are zero

Converting an Octal Fraction to a Hexadecimal Fraction

This method uses octal arithmetic.

1. Select a units rod and set the octal fraction onto the rods to the right of the unit rod
2. Multiply each digit of the octal fraction by 16 (octal 20), working from left to right, by adding the octal double of the digit to the left neighbor rod and then clear the original rod
3. Take the overflow which has spilled over onto the units rod (and possibly also onto the left neighbor of the units rod) as a one or two digit octal number which must be mentally converted to a single hexadecimal digit which will be the next most significant digit of the hexadecimal fraction, then clear the units rod (and its left neighbor)
4. If there are remaining non-zero digits in the octal fraction, go to step 2
5. Else stop

Example: octal 0.55753 → hexadecimal 0.B7D6

00.55753 set the octal fraction onto the rods to the right of the units rod

12.05753 multiply 5 by octal 20 – double 5 (= octal 12) and add it to its left neighbor rod, then clear the 5

13.20753 multiply 5 by octal 20 – double 5 (= octal 12) and add it to its left neighbor rod, then clear the 5

13.36053 multiply 7 by octal 20 – double 7 (= octal 16) and add it to its left neighbor rod, then clear the 7

13.37203 multiply 5 by octal 20 – double 5 (= octal 12) and add it to its left neighbor rod, then clear the 5

13.37260 multiply 3 by octal 20 – double 3 and add it to its left neighbor rod, then clear the 3

00.37260 the overflow, 13 (octal 13 = hexadecimal B), is the next digit of the hexadecimal fraction (0.B000 so far) – clear the overflow

06.07260 multiply 3 by octal 20 – double 3 and add it to its left neighbor rod, then clear the 3

07.60260 multiply 7 by octal 20 – double 7 (= octal 16) and add it to its left neighbor rod, then clear the 7

07.64060 multiply 2 by octal 20 – double 2 and add it to its left neighbor rod, then clear the 2

07.65400 multiply 6 by octal 20 – double 6 (= octal 14) and add it to its left neighbor rod, then clear the 6

00.65400 the overflow, 7, is the next digit of the hexadecimal fraction (0.B700 so far) – clear the overflow

14.05400 multiply 6 by octal 20 – double 6 (= octal 14) and add it to its left neighbor rod, then clear the 6

15.20400 multiply 5 by octal 20 – double 5 (= octal 12) and add it to its left neighbor rod, then

clear the 5

15.30000 multiply 4 by octal 20 – double 4 (= octal 10) and add it to its left neighbor rod, then clear the 4

00.30000 the overflow, 15 (octal 15 = hexadecimal D), is the next digit of the hexadecimal fraction (0.B7D0 so far) – clear the overflow

06.00000 multiply 3 by octal 20 – double 3 and add it to its left neighbor rod, then clear the 3

00.00000 the overflow, 6, is the next digit of the hexadecimal fraction (0.B7D6 so far) – clear the overflow

stop – all octal digits are zero

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