

This is another modification to the Welton J. Crook square root algorithm. If you recall, that method uses a number which Crook calls the "root number" which is actually equal to twice the square root, less one. This "root number" is subtracted from the number whose root is to be found, or from its remainder, then incremented by two and subtracted again, and so on, until it cannot be subtracted any more. At this point, the next group of two digits in the remainder of the square is brought down, a zero is tacked onto the "root number", 11 is added to the "root number" and the process is repeated to find the next digit. This method has the advantage of not requiring division or multiplication, only subtraction and incrementation, but it is tedious and slow. I have previously developed a modification of this method to speed it up (see "Modification to the Crook square root algorithm" in the files section) by using a simple test to see whether the next digit is greater than or equal to five, and if so, to jump immediately to five by means of a single subtraction, skipping over the increment and subtract steps from one through five. This method requires maintaining a third number on the soroban; i.e., the actual square root, and may not be practical for smaller sorobans. With this in mind, I set out to find a way to speed up the Crook algorithm while only maintaining two numbers on the soroban. This new method does require simple multiplication, but avoids the necessity both for division (except for a single digit divided into a one or two digit number) and for revision downward if the digit estimate is too great.

Simply described, the method starts off like the original Crook algorithm, but each time a new group of two is brought down and 11 added to the "root number", we estimate how many times the "root number" can be subtracted from the remainder by (mentally) rounding up the first digit of the "root number" and dividing it into the first one or two digits of the remainder. This single digit quotient "q" is multiplied times the "root number" and the product subtracted from the remainder. A further amount is subtracted to make up for the fact that we didn't "grow" the "root number" by adding two for each time we subtracted it. After subtracting this further amount (equal to $q*(q-1)$), we adjust the "root number" by adding $2*(q-1)$ and continue with the Crook algorithm. If we undershot the value of "q", it will be corrected as part of the normal Crook algorithm. When we have as many significant figures as we need, we find the root by adding one to the "root number" and dividing it by two, just as in the original algorithm. This is most easily illustrated by example.

$$\text{SQRT}(3.141592653590) = 1.77245 \text{ to six significant digits}$$

Square (or remainder)	"Root Number"
3.14 15 92 65 35 90	

Subtract the largest possible square from the first group; i.e., $3 - 1$. The largest possible square was one, its root is 1, so set the "root number" equal to $2*1 - 1 = 1$

- 1

2

Bring down the next group, tack on a zero to the "root number" and add 11	1
2 14	21

In the original algorithm, we would subtract 21, then 23, 25,...etc., but here we will round up the first digit of 21 to 3 and divide that into the first two digits of 214 (21) to get 7 for the value of "q". We then subtract $7*21$:

$$\begin{array}{r} - 14 \\ - 07 \\ \hline 67 \end{array}$$

Now subtract $q*(q-1)$ or $7*6 = 42$ to make up for having subtracted 21 seven times, instead of having subtracted 21, 23, 25, ...etc.

$$\begin{array}{r} - 42 \\ \hline 25 \end{array}$$

And adjust the "root number" by adding $2*(q - 1) = 12$
33

The remainder 25 is less than 33, so:

Bring down the next group, tack on a zero to the "root number" and add 11

$$25 \ 15 \qquad \qquad \qquad 341$$

Round up the first digit of 341 to 4 and divide that into the first two digits of 2515 (25) to get 6 for the value of "q". We then subtract $6*341$:

$$\begin{array}{r} - 18 \\ - 24 \\ - 06 \\ \hline 4 \ 69 \end{array}$$

Now subtract $q*(q-1)$ or $6*5 = 30$

$$\begin{array}{r} - 30 \\ \hline 4 \ 39 \end{array}$$

And adjust the root number by adding $2*(q - 1) = 10$
351

Because the remainder 439 is greater than 351, we have undershot the value of q -- the easiest thing to do is continue with the standard Crook algorithm

Increment the "root number" by 2 and subtract it from the remainder:

$$\begin{array}{r} - 353 \\ \hline 86 \end{array}$$

86 is less than 353, so:

Bring down the next group, tack on a zero to the "root number" and add 11

$$86 \ 92 \qquad \qquad \qquad 3541$$

Round up the first digit of 3541 to 4 and divide that into the first digit of 8692 (8) to get 2 for the value of "q". We then subtract $2*3541$:

$$\begin{array}{r} - 6 \\ - 10 \end{array}$$

$$\begin{array}{r} - 08 \\ - 02 \\ \hline \end{array}$$

16 10

Now subtract $q*(q-1)$ or $2*1 = 2$

$$\begin{array}{r} - 02 \\ \hline \end{array}$$

16 08

And adjust the root number by adding $2*(q - 1) = 2$

3543

1608 is less than 3543, so:

Bring down the next group, tack on a zero to the "root number" and add 11

16 08 65

35441

Round up the first digit of 35441 to 4 and divide that into the first two digits of 1608 (16) to get 4 for the value of "q". We then subtract $4*35441$:

$$\begin{array}{r} - 12 \\ - 20 \\ - 16 \\ - 16 \\ - 04 \\ \hline \end{array}$$

1 91 01

Now subtract $q*(q-1)$ or $4*3 = 12$

$$\begin{array}{r} - 12 \\ \hline \end{array}$$

1 90 89

And adjust the root number by adding $2*(q - 1) = 6$

35447

19089 is less than 35447, so:

Bring down the next group, tack on a zero to the "root number" and add 11

1 90 89 35

354481

Round up the first digit of 354481 to 4 and divide that into the first two digits of 1908935 (19) to get 4 for the value of "q". We then subtract $4*354481$:

$$\begin{array}{r} - 12 \\ - 20 \\ - 16 \\ - 16 \\ - 32 \\ - 04 \\ \hline \end{array}$$

49 10 11

Now subtract $q*(q-1)$ or $4*3 = 12$

$$\begin{array}{r} - 12 \\ \hline \end{array}$$

49 09 99

And adjust the root number by adding $2*(q - 1) = 6$
354487

Because 490999 is greater than 354487, we have undershot the value of q --
continue with the standard Crook algorithm:

Increment the "root number" by 2:

354489
- 35 44 89

13 65 10

136510 is less than 354489, so we are finished, because even after dividing by two we will still have six digits, as desired. All we have to do is add one to 354489 and then divide by two, as in the original Crook algorithm, and then check to see whether we need to round the last digit up.

$354489 + 1 = 354490$. $354490/2 = 177245$.

To test if roundup is needed, we can use the same method I used in my earlier modified Crook algorithm: Tack on 25 to the end of the square root (to get the accumulated number which would be subtracted from the remainder if the next digit were five) and compare that number with the remainder above (with the next two digits brought down). Because the remainder, 13,651,090 is less than 17,724,525, the next digit will be less than five and we do not need to round up. The final answer, to six digits, is 1.77245.

I hope you have found this to be both interesting and useful.

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