Easy Division Method for Soroban/Abacus

Here is an easy method for division on the soroban or abacus (perhaps better suited to sorobans as they typically have more rods than the Chinese suan pans). The method was developed in an attempt to make division easy by reducing mental multiplication as much as possible in favor of comparison and subtraction and also by the simplification of quotient estimation. The method does require an extra step of computing the divisor multiplied by five, but this is a trivial calculation, as described in Note 2 below.

The method is fundamentally long division with repeated subtraction of the divisor from the current working dividend fragment, but with improvements over the totally brute force method. Having available the 5x multiple of the divisor means one subtraction can replace five for quotient digits of five or greater. Equally important, it ensures that for quotient digit estimation (deciding how many times the divisor will divide into the current dividend fragment), after subtracting the 5x multiple of the divisor if necessary, we never need to consider any multiple higher than four times the divisor. This also makes it easier to make the estimation by using an “estimation divisor” which is greater than the actual divisor and for which it is very easy to mentally compute the 2x, 3x, and 4x multiples, while at the same time ensuring that we never undershoot the quotient digit by more than 1. An additional benefit is that having the 5x multiple of the divisor, we also have available at a glance one half of the divisor which makes it a trivial matter to round off the quotient to a certain number of significant figures.

This method compared to other methods of division on the abacus

Pro:

- It requires less mental effort than any other method I know of – no special division tables to memorize, and you don’t even need to do any multiplication other than to mentally multiply the one digit (2 – 9) or two digit (12, 15, or 25) estimation divisor by 2, 3, or 4.

- Revision, when required, is always to revise the quotient upward and subtract from the dividend, and revision will never be required more than once per quotient digit.

- It is reasonably fast.

- Rounding off the quotient is easy.

- It is a simple method to apply and to remember for those who only occasionally perform division on the abacus.

- It is applicable to all divisors and dividends – the amount of revision required does not strongly depend on the particular numbers being divided.

Con:

- It is not quite as fast as some other methods – maybe not the best method when maximum speed is needed.

- It requires an extra step to compute five times the divisor (but this is very easy).

- Extra rods are needed to hold the 5x multiple of the divisor, so it is better suited to the soroban than the suan pan as sorobans usually have more rods than suan pans.

The procedure after setting the divisor and dividend and calculating the 5x multiple of the divisor is, as in normal long division, to begin with the first dividend fragment which is greater than or equal to the divisor. Now compare this fragment to the 5x multiple of the divisor; if it is larger than the 5x multiple, set the appropriate quotient digit to five and subtract the 5x divisor multiple from the dividend fragment to obtain a new dividend fragment. Now that we know that the divisor will divide into our current dividend fragment a maximum of four times, instead of trying to do mental division with the actual divisor to estimate the quotient digit (to be added to the zero or five already set), it is easier to use a number greater than the divisor but which is much easier to mentally multiply by two or three or four. Using a number greater than the divisor ensures that the quotient digit we estimate will never be too large and we will never subtract too much from the dividend fragment and thus get a negative number. By careful choice of the number we use for the estimation divisor we can also ensure that we never undershoot the quotient digit by more than one. The divisor (normalized to between one and ten) will fall into one of the ranges in the table below. For a divisor in a particular range, use the associated estimation divisor. For example, if the divisor is 372.6, test whether 4 will divide into the first two digits of the dividend fragment once, twice, three times, four times, or not at all. Add this number to the 0 or 5 already set for the quotient digit and subtract the divisor from the dividend fragment that many times. Then check to see if the dividend fragment is now less than the divisor. If not, increment the quotient digit and subtract the divisor once more. When the dividend fragment has been reduced to less than the divisor, repeat the process for successive quotient digits.

These instructions can be condensed into a simple algorithm. Assume we start with the problem set with the divisor on the left, the dividend in the middle, and five times the divisor on the far right end of the soroban with all other rods zeroed. Assume the divisor has ‘d’ digits, and the associated estimation divisor has ‘e’ digits (e will be either 1 or 2). Start by selecting the current quotient digit position to be the second rod to the left of the most significant digit of the dividend.

1. Compare the divisor to the ‘d+1’ digits group just to the right of the current quotient digit position (rightmost digit of this group aligned with the least significant digit of the divisor). If this digit group (dividend fragment) is greater than or equal to the divisor, go to step 3. Else go to step 2.

2. Select the new current quotient digit position to be one rod to the right of the previous one and select the new dividend fragment to be the ‘d+1’ digits to the right of the new current quotient digit position. Go to step 1.

3. Compare the divisor multiplied by five to the ‘d+1’ digit dividend fragment. If the fragment is greater than or equal to five times the divisor, subtract five times the divisor from the fragment and add five to the current quotient digit. Go to step 4. If the fragment was less than five times the divisor, go to step 4.

4. The current quotient digit at this point is either 0 or 5. We must now use the estimation divisor to find how much to add to this. Compare the estimation divisor to the ‘e+1’ digit number just to the right of the current quotient digit position to determine the greatest integer number of times that the estimation divisor will divide into the ‘e+1’ digit number. Add this integer to the current quotient digit (0 or 5) and subtract the divisor that many times from the ‘d+1’ digit dividend fragment. The one exception to this rule is when the first digit of the divisor is 9, making the estimation divisor equal to 10 (or 1) – in this case no division estimation is necessary; just use the digit following the current quotient digit as the estimated quotient digit to be added to the current quotient digit and subtract the divisor that many times from the ‘d+1’ digit dividend fragment. Go to step 5.

5. Compare the divisor to the remaining ‘d+1’ digit dividend fragment. If the divisor is greater than the fragment, go to step 2. If the divisor is less than or equal to the fragment, revision is needed. Subtract the divisor from the fragment and add one to the current quotient digit. No further revision will be needed, so go to step 2.

Continue the above process until either the remaining dividend is zero (if the dividend was an integer multiple of the divisor), or until the desired number of quotient digits have been obtained, in which case check to see if the last quotient digit needs to be rounded upward. If the ‘d+1’ digit fragment to the right of the <u>next</u> quotient digit to be found is larger than or equal to the divisor multiplied by five, increment the last quotient digit to round off the quotient. If the fragment is smaller than the 5x multiple of the divisor, rounding up is not needed.

For divisors between	Use this estimation divisor
1 to 1.2	1.2 (12)
1.2 to 1.5	1.5 (15)
1.5 to 2	2
2 to 2.5	2.5 (25)
2.5 to 3	3
3 to 4	4
4 to 5	5
5 to 6	6
6 to 7	7
7 to 8	8
8 to 9	9
9 to 10	10 (or 1) – special case (see step 4 above)

Derivation of these numbers is described in Note 3 below.

A few examples will help in understanding the method.

Example 1: 8710.8 / 11.9 = 732

Set the problem – divisor on left end of the soroban, dividend in the middle, 5x the divisor on the right end. Estimation divisor = 12, d = 3, e = 2

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          U                 place dividend left of a unit rod by 4-2-2 = 0 rods
                                (dividend digits – divisor digits - 2)
119xxxxx0087108xxxxxx595    current quotient digit location two rods left of 8
         -595               subtract 5x divisor from 0871 and set quotient digit=5
119xxxxx5027608xxxxxx595
                            estimation divisor is 12 and divides into fragment 027
                                twice, so add 2 to quotient and subtract divisor
       +2                       twice from dividend fragment 0276 (subtract 1
         -119                   twice, 1 twice, then 9 twice)
         -119
119xxxxx7003808xxxxxx595    finished with this quotient digit (0038 < 119)
                            new dividend fragment 0380 is less than 5x divisor, no
                                5x subtraction
                            estimation divisor 12 divides into 038 3 times, so set
         3                      quotient digit = 3 and subtract divisor 3 times
          -119                  (subtract 1 (3 times), 1 (3 times), 9 (3 times))
          -119
          -119
119xxxxx7300238xxxxxx595    finished with this quotient digit (0023 < 119)
                            new dividend fragment 0238 is less than 5x divisor, no
                                5x subtraction
                            estimation divisor 12 divides into 023 once, so set
          1                     quotient digit = 1 and subtract divisor
           -119
119xxxxx7310119xxxxxx595    undershot quotient digit by 1, revise
         +1
           -119
119xxxxx7320000xxxxxx595    finished – least significant digit of quotient is on
          U                     the unit rod
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Example 2: 8091 / 93 = 87

This example shows how when the divisor begins with nine, thus making the estimation divisor equal to ten (or one), the rule of dividing the ‘e’ digit estimation divisor into the ‘e+1’ digits immediately following the current quotient digit no longer applies. Instead, just use the first digit of the dividend fragment following the current quotient digit as the estimated current quotient digit to add to the 0 or 5 already determined.

Set the problem - divisor on left end of the soroban, dividend in the middle, 5x the divisor on the right end. Estimation divisor = 1, d = 2, e = 1

Special case – no estimation division needed, just use the first digit of the dividend fragment as the estimated quotient

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        U                   place dividend left of a unit rod by 4-2-2 = 0 rods

                            current quotient digit location two rods left of 8
93xxxx008091xxxxx465        93 doesn’t go into 080, so add a digit to the
                               dividend fragment and move the current quotient
                               digit location one rod to the right
93xxxx008091xxxxx465        dividend fragment 809 is greater than 5x divisor
       5                    set quotient digit  = 5
       -465                 subtract 5x divisor
93xxxx053441xxxxx465
                            estimation divisor 1 divides into 3 (first digit of
                               dividend fragment) 3 times, or, more simply, just
                               use 3 as the estimated quotient and add it to the
     +3                        5 and subtract the divisor 3 times
        -93
        -93
        -93
93xxxx080651xxxxx465        finished with this quotient digit (065 < 93)
                            new dividend fragment 651 is greater than 5x divisor
        5                   set quotient digit = 5
        -465                subtract 5x divisor
93xxxx085186xxxxx465
                            again, use first digit of fragment (1) as the estimated
       +1                      quotient, add it to the 5 and subtract the divisor
         -93
93xxxx086093xxxxx465        dividend fragment = divisor, revision needed
       +1
         -93
93xxxx087000xxxxx465        finished – least significant digit of quotient is on
        U                      the unit rod
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Example 3: 2908108 / 734 = 3962

Set the problem – divisor on left end of the soroban, dividend in the middle, 5x the divisor on the right end. Estimation divisor = 8, d = 3, e = 1

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          U                 place dividend left of a unit rod by 7-3-2 = 2 rods

                            current quotient digit location two rods left of 2
734xxx002908108xxxxx3670    734 doesn't go into 0290, so add a digit to the
                                dividend fragment and move the current quotient
                                digit location one rod to the right
734xxx002908108xxxxx3670    dividend fragment 2908 is less than 5x divisor,  no
                                5x subraction
                            estimation divisor 8 divides into 29 3 times, so set
                                quotient digit = 3 and subtract divisor 3 times
       3
        -734
        -734
        -734
734xxx030706108xxxxx3670    finished with this quotient digit (0706 < 734)
                            new dividend fragment 7061 is greater than 5x divisor
        5                   set quotient digit = 5
        -3670               subtract 5x divisor
734xxx035339108xxxxx3670
                            estimation divisor 8 divides into 33 4 times
       +4                   add 4 to the quotient digit
         -734               subtract the divisor 4 times
         -734
         -734
         -734
734xxx039045508xxxxx3670    finished with this quotient digit (0455 < 734)
                            new dividend fragment 4550 is greater than 5x divisor
         5                  set quotient digit = 5
         -3670              subtract 5x divisor
734xxx039508808xxxxx3670    estimation divisor 8 divides into 08 1 time
        +1                  add 1 to the quotient digit
          -734              subtract the divisor once
734xxx039601468xxxxx3670    finished with this quotient digit (0146 < 734)
                            new dividend fragment 1468 is less than 5x divisor
                            estimation divisor 8 divides into 14 one time
          1                 set quotient digit = 1
           -734             subtract divisor once
734xxx039610734xxxxx3670    dividend fragment > or = divisor indicates undershoot,
         +1                 revise
           -734
734xxx039620000xxxxx3670    finished – least significant digit of quotient is on
          U                     the unit rod
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Rounding off:
In the above example, if we only wanted the quotient to three significant digits, we could have rounded off the solution after computing just three quotient digits. Back up to the point where we have just determined the third quotient digit (6):

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734xxx039601468xxxxx3670
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To determine whether or not the quotient should be 3960 or rounded up to 3970, we just need to compare the next dividend fragment, 1468, to the 5x divisor multiple, 3670 (actually we are comparing the remainder 146.8 to half the divisor 367.0). Because 1468 is less than 3670, we don't round up the third digit of the quotient, so the solution, to three significant digits, is 3960.

Note 1. When doing multiple subtractions of the divisor, I find it easier to perform all the subtractions on one rod, then move to the right neighbor rod and do all the subractions, then the next rod, etc. So, for example, if I had to subtract 734 three times, I would subtract 7 three times, then move right one rod and subtract 3 three times, then move right again and subtract 4 three times. This seems both easier and faster to me, but of course you can subtract 734 three times if you prefer. One way to shorten the process of multiple subtractions which can sometimes be used, is to mentally double the divisor and subtract the double; this way, two subtractions can be reduced to one, three subtractions to two, and four subtractions to two. For long divisors which are not easily mentally doubled, it's probably best just to do all the subtractions.

Note 2. Calculation of the divisor multiplied by five: place the divisor at the far right end of the soroban, except for the very right-most rod which remains cleared to zero. Using this far right-most rod as the least significant digit, we have multiplied the divisor by ten and now must divide by two. Starting at the right-most rod, proceed as follows:

1. If the number on the rod is even, go to step 2, else decrement the number to make it even and add five to the right neighbor rod by moving its heaven bead to the counter beam. Now go to step 2.
2. Subtract half of the count on the rod
3. Move one rod to the left and repeat steps 1 and 2 – continue until all non-zero rods have been processed – we now have 5x the divisor at the right end of the soroban.

Note 3. Determination of "estimation divisors"

The object is, for any divisor, to find a one or two digit number larger than the corresponding digits of the actual divisor to use as a trial divisor to determine an estimated quotient digit. The estimation divisor should be a number whose 2x, 3x, and 4x multiples can be mentally computed without effort. Using an estimation divisor larger than the actual divisor ensures that we will never overshoot the quotient digit, so when we multiply the actual divisor by the estimated quotient digit we will never have a product larger than the current dividend fragment from which this product is to be subtracted. We will never have to revise the quotient digit downward, but of course, we may need to revise upward. But we don't want to need revision by more than one, as each revision requires another subtraction of the divisor from the dividend fragment. The problem is, for any given divisor, to determine the best estimation divisor.

We know that, having already subtracted, if necessary, five times the divisor from the dividend fragment, the actual divisor and estimation divisor will only divide into the dividend fragment a maximum of four times. How much larger than the actual divisor can we make the estimation divisor without danger of estimating a quotient digit smaller than the actual quotient digit by more than one? Let d = the actual divisor, e = the estimation divisor, q = the actual quotient digit, and D = the dividend fragment. The worst case situation is when the divisor divides into the dividend fragment exactly q times: D = q*d. How much larger can e be relative to d so that e divides into the dividend fragment exactly q-1 times (so that if e were any larger, we would estimate the quotient digit to be q-2); that is to say, where D = (q-1)*e ?

We have D = q*d and D = (q-1)*e, so q*d = (q-1)*e or e = d*(q/q-1). We know that q is a maximum of four, so we could have e = (4/3)*d, or (3/2)*d, or (2/1)*d. The smallest of these is 4/3 and this will ensure that we won't undershoot the quotient by more than one. Note that if we hadn't first subtracted the 5x multiple of the divisor (when necessary) the quotient digit could be any number up to nine and we would be limited to an estimation divisor of no more than 9/8 of d to insure that we would never undershoot the true quotient digit by more than one.

So now that we know that our estimation divisor can be a maximum of 4/3 the actual divisor without undershooting the quotient digit by more than one, how do we choose a good set of estimation divisors? We could start at 1 and just multiply by 4/3, so the estimation divisor for all divisors between 1 and 1.333... would be 1.333..., for all divisors between 1.333... and 1.777… would be 1.777…, etc. These are not numbers for which we can easily compute the 2x, 3x, and 4x multiples, but we can pick any smaller number, so let’s use these estimation divisors which are easy to multiply by 2 or 3 or 4 and are still less than or equal to 4/3 times the minimum actual divisor in the corresponding range (all normalized to between one and ten – shift the decimal point as required by the problem):

For divisors between	Use as estimation divisor	Est. divisor/min. divisor	
1 and 1.2	1.2 (or 12)	1.2/1	= 1.20
1.2 and 1.5	1.5 (or 15)	1.5/1.2	= 1.25
1.5 and 2	2	2/1.5	= 1.333…
2 and 2.5	2.5 (or 25)	2.5/2	= 1.25
2.5 and 3	3	3/2.5	= 1.20
3 and 4	4	4/3	= 1.333…
4 and 5	5	5/4	= 1.25
5 and 6	6	6/5	= 1.20
6 and 7	7	7/6	= 1.166…
7 and 8	8	8/7	= 1.143…
8 and 9	9	9/8	= 1.125
9 and 10	* 10 (or 1)	10/9	= 1.111…

Single digit estimation divisors will be divided into the two-digit number just to the right of the current quotient digit position and two-digit estimation divisors (12, 15, and 25) will be divided into the three-digit number just to the right of the current quotient digit position. All of these estimation quotients will have an integer part of 0, 1, 2, 3, or 4.

* Exception to the rule – for divisors between 9 and 10, using an estimation divisor of 10, the rollover to a new decade for the estimation divisor means the dividend fragment must be shifted one digit before dividing the estimation divisor into it. The net effect is that we divide the 10 into the two-digit number just to the right of the current quotient digit position (instead of into the three-digit number), or, equivalently, we can use 1 as the estimation divisor and divide that into the single-digit number just to the right of the current quotient digit position. But this is the same as simply using the single-digit number to the right of the current quotient digit position as the new current quotient digit to be added to the 0 or 5 previously determined and no estimation division is needed.

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v3 – typos corrected - June, 2021
v4 – added exceptional case for divisor with first digit of 9, added another example to illustrate this - June, 2021