Calculating Logarithms on a Soroban - Contributed by Deji Adegbite

I've been working on how I can extract the logarithms of numbers on a Soroban. I came up with the following technique. Please reply me and tell me what you think. The technique is still in its infant stage. I'd like you to help me fine-tune it.

Actually I'm still looking for ways to take it further. Now I will be introducing you to it. I will use logarithm of 6 (base 10) as example and I will explain. Please I'd like comments, suggestion, and further ideas. You can introduce it to as many soroban fans as possible.

- Set up 10 on the far right end of the soroban.
- Find the Square root of 10. Clear off the remainder once you get its square root to 3 decimal places.
- On the left side of the Soroban, 2 rods from the end, set up 1.
- About 6 rods to the right of that 1(on the left), set up 1.
- Now you should have 1 and 1 on the left side of the soroban and 3.162 (the square root of 10) on the right hand side.
- Extract the square root of 3.162 without disrupting the one that is presently there. This means setting up another 3.162 and getting the square root to 3 decimal places.
- Remember to clear off the remainder after calculating the square root.
- The square root from the last step should be 1.778.
- Multiply 3.162 by 1.778 without disrupting any of both numbers (3.162 and 1.778).
- The result from the last step should be 5.622 (remember to stop after you get the answer to 3 decimal places).
- Now, on the left hand side of the soroban, take the 1 that is further from the left edge, double it and add 1 to the result (this should give us 2 x 1 = 2; 2 + 1 = 3).
- Now, add 1 to the 1 that is close to the left edge.
- Now, your soroban should show 2 and 3 (respectively) on the left side and 5.622 and 1.778 (in any order) on the right side.
- Get the square root of 1.778 (to 3 decimal places). It doesn't matter if the 1.778 gets removed in the process, we don't need it again. Remember to clear the remainder when you're done.
- The result from the last step should give us 1.333.
- Multiply 1.333 by the 5.622. Remember to keep the 1.333 and 5.622 intact.
- The answer should give us 7.494 (remember it is to 3 decimal places).
- On the left hand side of the soroban, double the 3 to get 6 and add 1 to the 2 to get 3.
- Now your soroban should show 3 and 6 on the left hand side (respectively), and 1.333, 5.622, 7.494 (in any order) on the right hand side.
- Clear off the 7.494 and find the square root of 1.333 to 3 decimal places. Once again, don't worry if the 1.333 gets wiped off.

Remember to clear off the remainder when you're done.

- The square root of 1.333 is 1.154.
- Multiply the 5.622 by the 1.154 to get 6.487.
- On the left hand side of the soroban, multiply the 6 by 2 to get 12 and add 1 to the 3 on the left.
- Now, your soroban should show 4 and 12 on the left hand side, and 6.487, 5.622, and 1.154 on the right.
- Clear off the 6.487.
- Find the square root of 1.154 to 3 decimal places. This should give 1.074.
- Multiply the 5.622 by the 1.074 to get 6.038. Remember, we need only 3 decimal places.
- On the left hand side of the soroban, multiply the 12 by 2 to get 24 and add 1 to the 4 to get 5.
- Now, we should have 5 and 24 on the left hand side, and (in any order) 5.622, 1.074, and 6.038 on the right side.
- Clear off the 6.038.
- At this point, look at the 1.074. Because there is a zero between the 1 and the 74, simply half the 74 to get 1.037. This is the Square root of 1.074.
- Now multiply the 5.622 by the 1.037. This gives 5.830 – to 3 decimal places.
- Move to the left hand side of the soroban, double the 24 and add 1. This gives 48 + 1 = 49.
- Add 1 to the 5 to give 6.
- Now your soroban should show 6 and 49 on the left side, and 1.037, 5.622 and 5.830 on the right hand side.
- Clear off the 5.622.
- Get the square root of 1.037 once again, simply half the 37 part to get the square root i.e. 1.018.
- Multiply the 5.830 by 1.018 to give 5.934.
- On the left side of the soroban, double the 49 and add 1. This gives 99.
- Add 1 to the 6 to give 7.
- Now, you should have 7 and 99 on the left and 5.934, 5.830 and 1.018 on the right. Remember that the numbers on the right can be in any order.
- Clear off the 5.830.
- Get the square root of 1.018. Remember, just half the 18 part to give 1.009.
- Multiply the 5.934 by the 1.009 to get 5.987.

- Multiply the 99 on the left by 2 and add 1 to give 199.
- Add 1 to the 7 to get 8.
- Clear off the 5.934.
- Get the square root of 1.009. This gives 1.004.
- Multiply 5.987 by 1.004 to get 6.010.
- On the left side of the soroban, double the 199 to give 398 and add 1 the 8 to get 9.
- Clear off the 6.010 and get the square root of 1.004 to get 1.002.
- Multiply 5.987 by 1.002. This gives 5.998.
- Once again on the left hand side of the soroban, multiply 398 by 2 and add 1. This gives 797 and add 1 to the 9 to get 10.
- Clear 5.987 and get the square root of 1.002 to get 1.001.
- Multiply 5.998 by 1.001 to give 6.003.
- Double 797 to get 1594 and add 1 to the 10 to get 11.
- We're almost there. Clear off all the numbers to the right and leave only the ones on the left i.e. you'll leave only the 1594 and the 11.
- Now, we're going to raise 2 to the power of 11 to get 2048. Then we'll divide 1594 by 2048 to get the log of 6. Don't worry, there's an easy way to do this just complete the remaining steps.
- Divide 1594 by 2 to get 797.
- Now take 1 from the 11 to get 10.
- Divide 797 by 2 to get 398.5 and take 1 from the 10 to get 9.Now divide 398.5 by 2 and take 1 from 9.
- Now your soroban should display 199.25 and 8.
- Now keep dividing and subtracting until the 8 reduces to 0.
- After the 8 reduces to 0 all you should have left on your soroban should be the log of 6 to base 10.
- The end result should be 0.778 (remember – to 3 decimal places).

That is the logarithm of 6 (base 10). You can check with a calculator – mine gives log 6 = 0.778.

The technique is surprisingly accurate especially if you consider that you are using beads. It still needs to be refined and perfected. I'll explain the whole thing over again. Here's the outline.

1.	0		10	
2.	1	1	sqr root(10) = 3.162	
3.	1	1	3.162,	sqr root(3.162) = 1.778
4.	2	3	3.162 x 1.778 = 5.622	
5.	2	3	1.778	5.622
6.	2	3	sqr root(1.778) = 1.333	5.622
7.	3	6	1.333 x 5.622 = 7.494	
8.	3	6	sqr root(1.333) = 1.154	5.622
9.	4	12	1.154 x 5.622 = 6.487	
10.	4	12	sqr root(1.154) = 1.074	5.622
11.	5	24	1.074 x 5.622 = 6.038	
12.	5	24	sqr root(1.074) = 1.037	5.622
13.	6	49	1.037 x 5.622 = 5.830	
14.	6	49	1.037	5.830
15.	6	49	sqr root(1.037) = 1.018	5.830
16.	7	99	1.018 x 5.830 = 5.934	
17.	7	99	1.018	5.934
18.	7	99	sqr root(1.018) = 1.009	5.934
19.	8	199	1.009 x 5.934 = 5.987	
20.	8	199	1.009	5.987
21.	8	199	sqr root(1.009) = 1.004	5.987
22.	9	398	1.004 x 5.987 = 6.010	
23.	9	398	1.004	5.987
24.	9	398	sqr root(1.004) = 1.002	5.987
25.	10	797	1.002 x 5.987 = 5.998	
26.	10	797	1.002	5.998
27.	10	797	sqr root(1.002) = 1.001	5.998
28.	11	1594	1.001 x 5.998 = 6.003	

The explanation of the steps are as follows.

1. Set up 10 on the right hand side of the soroban.
2. Extract the square root of 10 which is 3.162. On the left hand side of the soroban, set up 1 at 2 rods to the end of the soroban. Then about 6 rods away, set up another 1.
3. Set up another 3.162 and find its square root which is 1.778
4. Multiply the 3.162 by the 1.778 to get 5.622. Move to the left hand side, because the product we just got is less than 6 (the number whose logarithm we're looking for), take the 1 on the left hand side of the soroban double it and add 1. This gives us 3 (2 x 1 = 2; 2 +1 = 3). Add 1 to the 1 that is nearer to the left edge of the soroban to give 2.
5. Clear off the 3.162.
6. Get the square root of 1.778 to get 1.333
7. Multiply the 1.333 by the 5.622. This gives us 7.494 Remember to still leave 1.333 and the 5.622 intact. The product is greater than 6 (the number whose log we want to find), double the 3 on left hand side to get 6 (don't add 1 after doubling because 7.494 is greater

than 6). Then, add 1 to the 2 to get 3

8. Once again, since 7.494 is greater 6, clear it off and get the square root of 1.333 which is 1.154. Remember that we need the square root only to 3 decimal places and also remember to clear off the remainder.

9. Multiply the 1.154 by 5.622. This gives 6.487. Always remember to leave the 2 numbers that you are multiplying intact. 6.487 is greater than 6 so just double the 6 to get 12 (don't add 1 after doubling) and add 1 to the 3 to get 4.

10. Get the square root of the 1.154 to get 1.074

11. Multiply the 1.074 by the 5.622 to get 6.038 which is greater than 6 (the number whose log we are trying to find), we will double the 12 (we'll not be adding 1) to get 24. Add 1 to the 4 to get 5

12. Clear the 6.038 and get the square root of 1.074. At this point here is how we will calculate the square root. Because there is a 0 between the 1 and the 74, just half the 74 part to get the square root. With this technique, we will be saying that the square root of 1.074 is 1.037. Thus, you don't have to set up anything again. Mentally half the 74 part.

13. Multiply the 5.622 by the 1.037. This gives us 5.830. Notice that this product is less than 6 so this time, after doubling the 24 we well add 1. This gives us 24 x 2 = 48; 48 + 1 = 49. Now add 1 to the 5 to get 6.

14. Now clear off the 5.622.

15. Get the square root of 1.037. Once again, simply half the 37 part to get the answer 1.018 which is the square root of 1.037.

16. Multiply the 5.830 by the 1.018 to get 5.934. Notice that the product is getting closer to 6. Since 5.934 is less than 6, we will double the 49 on the left and add 1. This gives us 99. Now add 1 to the 6 to get 7.

17. Clear off the 5.830.

18. Get the square root of 1.018. Remember to just half the 18 part to get the square root i.e. 1.009.

19. Multiply the 1.009 by the 5.934 (remember to keep both the 1.009 and the 5.934 intact). This gives us 5.987 which is less than 6 (the number whose log we are trying to find). Since the product 5.987 is less than 6, double the 99 and add 1. This should give us 2 x 99 = 198; 198 + 1 = 199. Now, add 1 to the 7 to get 8.

20. Clear off the 5.934.

21. Get the square root of 1.009 to get 1.004. Remember to just half the 009 part to get 004.

22. Multiply 5.987 by 1.004 to get 6.010. Now since 6.010 is less than 6, double the 199 on the left to get 398 (don't add 1 after doubling because the product we got 6.010 is greater than 6). Now add 1 to the 8 to get 9.

23. Now clear off the 6.010.

24. Get the square root of 1.004 to get 1.002.

25. Multiply 5.987 by 1.002 to get 5.998. Since 5.998 is less than 6, double the 398 on the left and add 1. This gives us 797. Now add 1 to the 9 to get 10.

26. Clear off the 5.987.

27. Now, get the square root of 1.002 to get 1.001.

28. Multiply 1.001 by 5.998 to get 6.003 which is too large so we'll double the 797 without adding 1. This gives us 1594. Now add 1 to the 10 to get 11.

29. Now clear off all the numbers to the right hand side of the soroban and leave only the ones on the left i.e. 11 and 1594.

We're there. To get the log of 6 (base 10), raise 2 to the power of 11 (2^11 = 2048). Now divide 1594 by the result – i.e. 1594/2048. Don't get scared. There is an easy way to do this. Just continue with the following steps.

- Divide 1594 by 2 to get 797. Now subtract 1 from 11 to get 10
- Divide 797 by 2 to get 398.5 and subtract 1 from 10 to get 9.
- Divide 398.5 by 2 to get 199.25 and subtract 1 from 9 to get 8.
- Continue until you can no longer subtract 1. That means till you get to 0.
- Once you're done, approximate the result (0.7783203125) to 3 decimal places (0.778) that's the log of 6 to the base of 10.

The last few steps are listed below.

i) 11 1594

ii) 10 1594/2 = 797

iii) 9 797/2 = 398.5

iv) 8 398.5/2 = 199.25

v) 7 199.5/2 = 99.625

vi) 6 99.625/2 = 49.8125

vii) 5 49.8125/2 = 24.90625

viii) 4 24.90625/2 = 12.453125

ix) 3 12.453125/2 = 6.2265625

x) 2 6.2265625/2 = 3.11328125

xi) 1 3.11328125/2 = 1.556640625

xii) 0 1.556640625/2 = 0.7783203125

That's all about the technique though it still needs to be fine-tuned and perfected. Feel free to contact me if you don't understand it. As you can see, it is quite accurate for a simple machine like the soroban. Remember to introduce it to as many soroban fans out there as you possibly can. I still need more ideas, comments and suggestion – don't forget to ask questions too. Some other ideas I need are; how I can reduce the number of rods used, setting up the soroban for the operation and what aspects of the technique can be done mentally. Also, as is obvious, the technique might not be very good for overtly high number bases (I tried get log 135 base 256). It may also not be very good for numbers with more than 1 decimal digit e.g. 1.563, 8.995, 3.95, etc.

Now just two more things –

First, My next research is on trigonometric ratios. That is, how to calculate the sine, cosine and tangent of angles on a soroban. I hope to get ideas and assistance from you guys out there. If there is anyone willing to assist me, let me know and I'll send you some ideas I've had so far though I'm still working on just the Sine of angles.

Secondly, I intend to write an article - "Interesting Abacus Techniques." I'm looking for people who will volunteer to place these articles on their websites. Please, if you are willing to assist me, just let me know. I will let you know how far I've gone as I compile the articles. I'll be so grateful.

The Soroban – Simple, yet intriguing.

- *Abacus: Mystery of the Bead*
- *Advanced Abacus Techniques*

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