Explanation of the Logarithm Technique - Deji Adegbite

Please print the page to make it easier to read

P.S So I'll be sure you understand the technique, please *reply me* and explain it to me again with another example.

*********Please get a pen and a piece of paper*********

Let me start from the very beginning.
We know that the logarithm of a number (N) is the power to which another number (the base) must be raised to get that number (N). For example, log 8 (base 2) = 3 i.e. you raise 2 to the power of 3 to get 8. Log 7 (base 49) = 1/2 i.e. you raise 49 to the power of 1/2 to get 7.

Now, lets imagine the logarithm scale as a straight line. Thinking in terms of base 10, we'll have 1 on the left end of the line and 10 on the right end of the line - remember it's in base 10. To get the logarithm of a number, what portion or fraction of the line would we cover to get to that number if we are moving from left to right? For example, if we we're looking for the logarithm of 3.162, i.e. the square root of 10, we would move half way across the line. Lets take another example. If the log scale was in base 16, we would have 1 on the left end of the line and 16 on the right end. Using this base 16 log scale, if we want to get the logarithm of 4, we would only have to move half way across the line to get to 4. This means the logarithm of 4 (base 16) is ½. Using the same base 16 scale, what is the logarithm of 8? ¾ or 0.75. Which means we have covered ¾ of the line by the time we get to 8 (moving from left - i.e. 1 - to right - i.e. 16). Using base 1000, what is the logarithm of 10? 1/3 or 0.333 i.e. we would have covered 1/3 of the line by the time we get to 10 once again moving from left (1) to right (1000 - the base).

Let's now imagine that our base 10 line is divided only into 2 parts. This means we would have 1 on the left end of the line, 3.162 half way across and 10 on the right end of the line. This also means that on this scale, log 1 = 0, log 3.162 = 1/2 and log 10 = 1. Now, let's divide the line into 4 parts. This means we'll have 1 on the left end, 1.778 ¼ way across, 3.162 half way across, 5.622 ¾ way across and 10 on the right end of the line. This means that log 5.622 = ¾ because it is ¾ way across the line and log 1.778 = ¼ because it is ¼ way across.

PS Don't continue if you don't understand. So I'll be sure you understand the technique, please reply me and explain it to me again with another example.

Get a pen and a piece of paper before you continue.

Draw a straight line on your piece of paper and lets imagine that we want to get the log of 5 to the base of 10 i.e. log 5 (base 10). Without dividing the line, we'll have 1 on the left end and 10 on the right end of the line. Let's divide the line into 2 equal parts. Now, we'll have 1 on the left end of the line, 3.162 half way across the line, and 10 on the right end of the line. If we used this scale, we'll be approximating the log of 5 to be ½ which isn't very accurate. We know that 5 is somewhere between 3.162 and 10. Therefore, to make our log scale more accurate, we'll divide the line into 4 equal parts. This means we'll have 1 on the left end of he line, 1.778 ¼ way across the line, 3.162 half way across the line, 5.622 ¾ way across and 10 on the right end of he line. With this scale, we will approximate the log of 5 to be 3.162 since 5.622 is too large i.e. 5.622 is greater than 5. Now we are approximating the log of 5 to be 0.5 or ½. Instead of doing that, we'll do it this way; the number of subdivisions before 3.162 is 2, and the log scale was divided into 4 parts, so, instead of saying ½, we'll say 2/4.

Now, let's further divide our log scale into 8 parts to make it more accurate. Now, we'll have 1.333 1/8 way across, 1.778 2/8 (or ¼) way across, 2.371 3/8 way across, 3.162 4/8 (1/2) way across, 4.216 5/8 way across, 5.622 6/8 (or ¾) way across, 7.498 7/8 way across and finally, we'll have 10. Now, we should have something closer to 5 on our log scale and that is 4.216 (4.216 is closer to 5 than 3.162) which is 5/8 way across. This means that using this scale, we would approximate the log of 5 to be 5/8. Now lets make our log scale more accurate by dividing it into 16 parts, this gives 1 on the left end of the line, 1.154 1/16 across, 1.333 2/16 (or 1/8) across, 1.539 3/16 across, 1.778 4/16 (or 2/8) across, 2.053 5/16 across, 2.371 6/16 (or 3/8) across, 3.651 9/16 across, 4.216 10/16 (or 5/8) across, 4.869 11/16 across, 5.622 12/16 (or 6/8) across, 6.493 13/16 across, 7.498 14/16 (or 7/8) across, 8.659 15/16 across, and 10 on the right end of the line.

Notice now that we have something closer to 5 than the previous result. The previous result gave us 4.216 which was 5/8 way across the line. Now we have 4.869 which is closer. Hence, we would approximate the log of 5 to be 11/16 (i.e. the log of 4.869).

So you see, to make our log scale more accurate, we keep dividing it. So the next time we divide our log scale, we'll divide it in 32 parts. Just continue the trend until you get as close to log 5 as you like but even if we stop here, we'll be approximating our log 5 to 11/16 = 0.687. My calculator tells me that log 5 = 0.698. This gives us a difference of 0.698 - 0.687 = 0.011. which is pretty close already!

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In the technique, we were incrementing the number on the far left of our soroban by 1. This is to help us keep track of how many times we have had to divide our log scale. In our last example, we divided 4 times. This means that there would be 16 subdivisions on our log scale. If we had divided one more time, we would have divided 5 times and we would have 32 subdivisions. This makes our log scale more accurate. Notice that if we divide our log scale t times, then we will have 2^t subdivisions. When we divided only once, we had 2^1 divisions i.e. 1 to 3.162 and 3.162 to 10.

For the rooting and multiplying part, here is the explanation;

Note that when marking our log scale, we don't just mark the line any how. For example, lets say you want to make a base 16 scale, how do you know what is supposed to be half way across the line? You get the square root of 16 first which gives us 4, then, mark half way across the line 4. Now lets divide the log 16 line again. This time, we would have 4 subdivisions with 1 on the left end, 4 half way across, and 16 on the right end of the line. What will be between 1 and 4? That will be ¼ way across the line, i.e. 16 raised to the power of ¼ which is 2. What will be between 4 and 16? Lets see…that's ¾ way across the line so that's 16 raised to the power of ¾ which gives us 8. Now your log scale (base 16) should have 1 on the left end, 2 ¼ way across, 4 ½ way across, 8 ¾ way across and 16 on the right end. Lets have another example. Lets say we have a line with base 10 000. At first, we had 1 on the left end and 10000 on the right end. When we divide the line into 2 parts, what would we have ½ way across? That would be 10000 raised to the power of half which gives 100. So now we would have 1 on the left end, 100 half way across, and 10000 on the right end. Lets say we divide this line again, this time into four equal parts. What would we have between 1 and 100? Notice that's now ¼ way across which means we'll have 10000 raised to the power of ¼ which gives us 10. What would we have between 100 and 10000? Hmm… that's ¾ way across the line and that gives us 10000 raised to the power of ¾ which is 1000. So your new log scale should now have 1 on the left end, 10 ¼ way across and 10000 on the right end.

I hope you're still following me! Let's say we have a base 256 log scale (I hope you still have your pen and paper) with which we want to get the log of 100 i.e. log 100 (base 256); Now let's draw our line and label it. This means we would have 1 on the left end and 256 on the right end. Now, lets divide the line into 2 equal parts. What would be ½ way across the line? That's 256 raised to the power of ½ which is 16. Now, if we stop at this point, we would be approximating the log of 100 to ½ since 16 is the nearest number to 100. We will not say 256 because 256 is greater than 100 and we're not considering greater numbers - only the lesser ones. Now lets divide the line into 4 equal parts. Where do you think the next value will fall on after 16? Don't think too far! The next value after 16 will be 64 i.e. 16 x sqr root(16) = 16 x 4 = 64. You don't believe? Well, lets see… what will be between 1 and 16? That's ¼ way across the line i.e. 256 raised to the power of ¼ which is 4. What will be between 16 and 256? That's 254 raised to the power of ¾ which is 64. At this point, we would approximate the log of 100 to ¾ since 64 is now nearer to 100. Please remember that we're not saying 256 because we're not considering larger numbers. Lets divide our log scale on more time. This means we will have 8 subdivisions. Remember we're sitting on the 64 mark (the number whose log we used as the log of 10). What will be between 64 and 256? That should be 64 x sqr root(4) = 64 x 2 = 128. How did I do that? By getting the square root of the previous unit length and then multiplying. In this case, the previous unit length was 4. The unit length is the next number after 1 on our log scale. Let's continue. We're still sitting on the 64 mark. Why? Because 128 (the number after 64) is greater than 100 and we are not considering higher numbers. So, we're still approximating the log of 100 to ¾? Yes but this time, remember we not have 8 subdivisions so we'll say 6/8 (double the numerator without adding 1 since our new approximation is still equal to our previous approximation). Now, another example. Let's get log 75 (base 10000). Draw your log line. We have 1 on the left end and 10000 on the right end. Now lets divide the line the first time. This means we have divided only once. How many subdivisions would we have? 2^1 = 2. What do we have ½ way between 1 and 10000? That's 10000 raised to the power of ½ which is 100. Our unit length is 100. That's the next number after 1. Also, at this point, we would approximate log 75 to be 0 i.e. log 1. We're not to approximate it to log 100 which is ½. This is because 100 is greater than 75 and we're not considering higher numbers. Now, lets divide the line a second time. This means we're have 4 subdivisions (since we're dividing the second time, we will have 2^2 subdivisions). What will we have between 1 and 100? Our new unit length is 10 i.e. the square root of the previous unit length which was 100. We're sitting on the 1 mark. So, what will be between 1 and 100, that's 1 x 10 = 10. If we use this scale which is still not very accurate, we would be approximating the logarithm of 75 to be equal to ¼. Why? Because 10 is the closest thing to 75. Why not 100? Because 100 is greater than 75 and we are not considering numbers that are greater. So, to make the scale more accurate, what do we do? We divide the line one more time. Note that this would be the third time we are dividing. So, how many subdivisions will we have on our log scale? That's 2^3 = 8. Why? Because this would be the third time we are dividing.

What would be our new unit length? That would be the square root of our previous unit length i.e. sqr root(10) = 3.162. We are sitting on the 10 mark i.e. in our previous approximation, we approximated the log of 75 to be equal to that of 10. So what will be between 10 and 100? Don't get your calculator yet! The answer will be the new unit length multiplied by the mark we were sitting on. In this case, 3.162 x 10 = 31.62. Now, we've moved a little closer! Remember we will not say 100 because we are not considering higher numbers. We're not sitting on the 31.62 mark. Now, what's the log? How many subdivisions do we have in total? That's 8 or 2 raised to the power of t where t is the number of times we have divided. How many subdivisions do we have before 31.62? 3. How do we get this? Observe that since 31.62 is still less than 75, the numerator in our previous answer is doubled and then 1 is added. In our previous approximation, we approximated the log of 75 to be ¼. The numerator is 1. So our new numerator would be 2 x 1 = 2; 2 + 1 = 3. So, our new approximation would be 3/8.

Let's stop at this point and take another example. I hope you still have your pen and paper.

What is the log of 7 to the base of 256?

Draw your straight line once again. We'll have 1 on the left end of the log scale, and we'll have 256 on the right end of the log scale. Let's divide the log scale the first time. How many subdivisions do we have? We've divided only once so the number of subdivision = 2^1 = 2. Now what is half way between 1 and 256? Hmm… that would be 256 raised to the power of ½ which is 16. At this point, what would be our approximation to log 7? 0/2 = 0. Its not ½ because that's 16 which is greater than 7 and we're not considering higher numbers. What is our unit length? That's 16 i.e. the next number after 1. So on what mark are we sitting? We are sitting on the 1 mark because we are approximating the log of 7 to be equal to that of 1.

Now, let's divide the line the second time. How many subdivisions will we have? That's 2^2 = 4. Remember that the number of subdivisions is 2^t where t represents how many times we have divide. What will be our new unit length? That's the square root of the previous unit length i.e. sqr root(16) = 4. What's the next number after 1? Remember we were sitting on the 1 mark in the first approximation. That's the new unit length multiplied by 1 - 1 x 4 = 4. What is the numerator of our answer? Double the previous numerator and add 1 i.e. 0 x 2 = 0; 0 + 1 = 1. We are adding the 1 because 4 is still less than 7. Remember this time we will be approximating our log of 7 to that of 4 since the next number after 4 (16) is greater than 7 and we are not considering greater numbers. How do we get the denominator? That's the total number of subdivisions which is 2^2 = 4. So the new approximation is ¼. Now, we are sitting on the 4 mark. Let's now divide the line 1 more time. How many subdivisions will we have now? We are dividing for the third time so that would be 2^3 = 8. This will be the new denominator of our answer. What is our new unit length? That's the square root of the previous unit length i.e. sqr root(4) = 2. So, from point 4 where we are sitting on, what will be between 4 and 16? That's 4 (because we're sitting on the 4 mark) multiplied by our new unit length which is 4 x 2 = 8. So, we will have 8 between 4 and 16. What will we approximate log 7 to? We will approximate it to that of log 4. Why? Because 8 is greater than 7 and we are not considering higher numbers. So, how do we approximate? Double the previous numerator to get the new one (don't add 1 after doubling because our approximation has not changed. We are still sitting on the 4 mark) i.e. 2 x 1 = 2. So our approximation will be 2/8 (not ¼). Let's continue. We'll divide for the last time then I'll stop. Let's divide our log scale 1 more time. How many subdivisions do we now have? We have divided 4 times now so the number of subdivisions will be 2^4 = 16. I hope you're still working with your pen and paper! What will be new unit length? That's the square root of the previous unit length i.e. sqr root(2) = 1.414. Remember we're still sitting on the 4 mark. So what will be between 4 and 8? That's 4 multiplied by the new unit length i.e. 4 x 1.414 = 5.656. This is closer to 7. So how do we approximate? 5.656 is less than 7 so we'll double the previous numerator and add 1 i.e. 2 x 2 = 4; 4 + 1 = 5. So, we'll have 5/16. We're not approximating the log of 7 to that of 8 because 8 is greater than 7 and we're not considering greater numbers. If we stop here, we would be implying that the log of 7 (base 256) is 5/16. So, you'll continue dividing and approximating till you get as close to log 7 as you like. You can try dividing up to 16 times. If you divide up to 16 times, how many subdivisions would we have? That's 2^16 = 65536. You see that would give us something very accurate. The more you divide, the more accurate the log scale becomes.

I hope you understand the whole thing. If you don't read it all over again until you do and mail me. Please use a pen and a piece of paper so you'll understand it better.

If you understand, you'll still have to mail me and explain it to me with another example so that I will know whether you understand it or not.

▪ *Calculating Logarithms on a Soroban*

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