Origin and Development of the Chinese Abacus*

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Introduction

Since the successive appearance of electric calculating machines and electronic digital computers, there may be some interest in tracing back to the origin and development of the earliest prototypes of computing devices. And among them, the Chinese abacus is preeminent in that:

(1) It is the oldest computer, and it is still in wide use.

(2) It is the simplest calculating device and yet complete by itself without the need of any ancillary or peripheral equipment.

(3) It is the first computing device, so far known, in which man has ever made use of the bi-quinary notation in its beads.

(4) It can be made of the simplest materials and by the simplest tools.

The Chinese abacus as a physical device for computing can be traced as far back as circa 1100 B.C. The development of the earliest prototype of the Chinese abacus dates at least as far back as the period between the sixth and third centuries B.C. In its latest general pattern [5, 7, 30a] it has existed for at least seventeen centuries. In China, Korea, Japan, South East Asian countries, and China South Sea islands, and to a lesser extent also in India and Russia, it has become an indispensable daily computing device in all banks, business institutions, commercial establishments, manufacturing plants, construction firms, and government agencies, in their business, sales, purchasing, cashier, accounting, auditing, estimating, fiscal, and control operations. At present it is being used by over half of the world's population.

Being entirely an external calculating device, bi-quinary in form but decimal in operation, the Chinese abacus does not need the drawing of a program nor the resort to a code for communication with it. Its first cost is the lowest among all digital computers, and it requires no maintenance charges. When skillfully operated with its ingeniously developed methods of calculation (which are concerned with arithmetical short-cuts rather than programming), it can hold its unique position as one of the world's most efficient computing devices.

Motivated by the development of modern high-speed computers, the Chinese abacus has been very frequently referred to in contemporary writings in the western world. But different authors in their fragmentary statements have attributed its development to diverse origins, none being free from historical speculation. The reason why its history has remained obscure to western authors lies in the fact that most of them could not read ancient Chinese works. Nor is its history familiar to the majority of modern Chinese mathematicians and engineers,

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for traditionally they have regarded this device as a common-place instrument because of its universal use. It is the desideratum of this paper to present the authentic origin of the Chinese abacus and its development.

The Chinese abacus was originated and developed independently of all others. It has no connection whatsoever with the abacus Pythagoricus using sand grains, the ancient bronze counting table or the abacus with beads moved in grooves used by the Romans [31], the ancient Peruvian abacus [26, 31], or the French abacus or universal reckoner [20, 21]. Nor does this paper attempt to trace any relation between the Chinese abacus and other minor versions [31] of the abacus such as "the s'choty" which is still in use in Russia, "the coulba" in Turkey, or "the choreb" in Armenia (now a part of U. S. S. R.).

Pre-Chinese-Abacus Computing Device

The genesis of physical computing devices in ancient China dates back to about 1100 B.C. in the early Chou Dynasty [1, 2, 3, 4, 6], insofar as authentic classics and dynastical books of history have recorded. There was then a high degree of civilization. The so-called "Six Arts"—ritual, music, archery, charioteering, writing, and mathematics—were the required fundamental branches of learning, with the climax in mathematics. Physical reckoning and computing devices were first recorded in two of the 56 chapters of *Yee Li*, or *The Book of Ritual* (social) *and Standards* (physical), one of the 13 ancient Chinese classics. Its original version was found by Lu Kung Wang (King of Lu) from Confucius' tomb mansion. "Yee" refers to ceremonial usages and ritual rules in the social sense, but it also means and includes standard physical devices, apparatus, or instruments adopted in usage, especially in connection with ceremonies. Both the chapter on "Sacrificial Bull Ceremony" and the chapter on "Grand Archery Ceremony" describe the then reckoning and computing device. It was made of bamboo (sometimes wood or ivory) in the form of digital rods, three to six inches long, of which a set consisted of as many as 271 rods [6b]. They were originally called either "calculating rods" or "counting rods", but the Chinese ancients used them for both counting and computing. With these digital rods operating methods were devised to perform addition, subtraction, multiplication, and division. Their application became widespread 'before' Confucius' time (551–478 B.C.) [4] and had remained in use but had gradually been completely replaced by the more efficient device, the abacus, until the early Sung Dynasty, around 1000 A.D. Later developments of the abacus mathematics were originated therefrom. The traditional use of bamboo rods as bar rails for carrying the beads of the Chinese abacus had its initiation in the bamboo calculating rods.

It must be noted here that the ancient Chinese calculating rods are different from Napier's bones [14, 16] (or rods) which were invented by the Scotch mathematician John Napier (1550–1617), also for computing. Napier's bones were taken to China during the seventeenth century by missionaries. There was no "word" on the ancient Chinese calculating rods, but "word" appeared on Napier's bones.

Advent of the Chinese Abacus, Chu Pan or Bead-Tray

The first improvement on the ancient Chinese calculating device consisted of the provision of a rectangular computing tray containing computing beads, made of finished wood as those of the Chinese abacus are. It was called *Chu Pan* [5, 30a, 30b], which marked the initial prototype of the Chinese abacus. Its origin was in the later part of the Chou Dynasty between the sixth and third centuries B.C. The rectangular tray was chess-board-like, divided into squares which had ten rows horizontally and as many columns as necessary. Each horizontal position denoted a cardinal numeral, 0 to 9, from bottom up. Each column represented a digit increasing from right to left. By placing beads in proper squares in the tray, numbers were represented for arithmetic operations. It resembles a second-quadrant cartesian system with digits as abscissae and cardinal numerals as ordinates. Figure 1 shows the number 23,975 represented in the bead tray at a particular instant of the calculation operations. The Chu Pan, or bead-tray, had since undergone evolutional improvements and had gradually replaced the ancient calculating rods.

Half-Size Chu Pan with Beads Having Two Colors

To reduce the size of the bead-tray to one half [5, 30] was the next improvement. This was accomplished by using beads of two different colors, yellow beads denoting numerals 0 to 4 from bottom up, and black ones representing 5 to 9 from top down, with digits also increasing from right to left. This scheme enabled

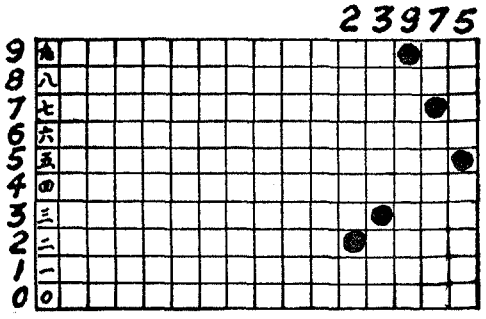

FIG. 1. Chu Pan, or Bead Tray

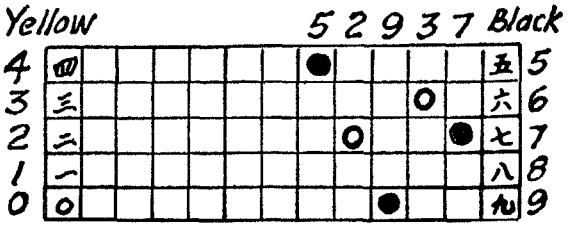

FIG. 2. Half-Size Tray with Beads in Two Colors

the bead-tray to be divided into only five horizontal rows of squares and as many vertical columns as before. Figure 2 shows the number 52,937 represented in a half-size Chu Pan at a particular instant of the calculation operations.

Authentic Accounts About the Chu Pan

In 213 B.C. Emperor Chin Shih Huang destroyed most of ancient Chinese books by fire. The progress of the Six Arts suffered an unprecedented loss. Ancient works on calculation methods and devices were mostly burnt. It was only through private preservation during the short Chin Dynasty and through the following state glorification of ancient classics during the Han Dynasty that such classics and other works had been handed down to later generations.

Near the close of the Later Han Dynasty, early in the third century, a book entitled *Collected Accounts of Mathematics Left by the Ancients* was written by Hsü Y'ao [5, 7, 8, 9, 30a, 30b], who first referred to the device of Chu Pan, or bead-tray, for computing. Though Hsü Y'ao in his later years became an official of the Wei Dynasty, he was a remnant of the closing period of the Later Han Dynasty. The extent to which Chu Pan was used by the Chinese ancients may be seen from Hsü Y'ao's verse referring to it as the device which "controls the four seasons and coordinates the three powers: heaven, earth, and man." This shows that it was used in calendar calculations, in geodetic determinations, and in computations relating to human affairs. And as will be described in the next section, the Chu Pan as revealed in Hsü Y'ao's work had at length assumed the same general pattern as that of the modern Chinese abacus except that beads had not yet been attached to the bar rails.

Hsü Y'ao's account has been regarded as most authentic by later writers, because his time was more proximate to the ancients. Scholars of the Ching Dynasty in the later eighteenth century once had some criticisms on other portions of Hsü Y'ao's entire work, but they could not invalidate, nor did they have sufficient evidences to disprove, the historical authenticity of Hsü Y'ao's account about the ancient Chu Pan.

Not only did Chu Pan represent the advent of the modern Chinese abacus, but also it must have originated at least as early as the later part of the Chou Dynasty between the sixth and the third centuries B.C., because that period was unquestionably considered as the latest ancient time in the days of Hsü Y'ao.

In the latter part of the sixth century, during the Northern Chou Dynasty, Ch'en Lan's *Commentary on Hsü Y'ao's Work* [7, 30a, 30b] further verified Chu Pan as precisely the prototype of the Chinese abacus.

In the early Tang Dynasty in the seventh century, Hsü Y'ao's work was formally incorporated into the then *Standard Arts and Literary Encyclopedia* [8]. This constituted the highest recognition of Hsü Y'ao's work at that time. Further, according to the *Civil Service Election Code* [9] promulgated by the Tang Dynasty, Hsü Y'ao's book was adopted as a required statutory text in addition to the other so-called "nine books and five treatises on mathematics."

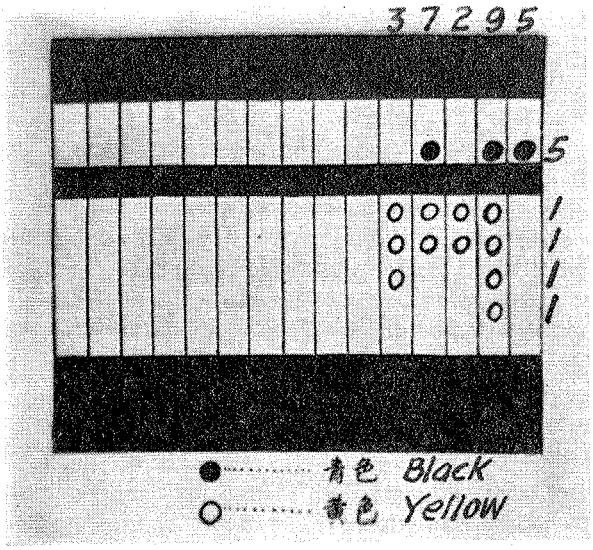

FIG. 3. The Abacus with Unattached Beads

Chinese Abacus with Unattached Beads

According to Ch'en Lan's Commentary on Hsü Y'ao's work referred to above, at least from the Han Dynasty before Hsü Y'ao's time, i.e., before the second century, the Chu Pan had almost exactly taken the form of the modern Chinese abacus but with unattached beads [5, 7, 30a]. It consisted of a wood board with top and bottom recesses for storing beads, and with rectangular column spaces in the middle part having an upper horizontal partition. In each column only five beads could be placed. One bead above the partition represented 5, while each (1, 2, 3, or 4) below it denoted only "one", with digits increasing from right to left as before. Thus the number 37,295 would be as indicated in figure 3 at a particular instant of the calculation operations. It took precisely the same pattern as the modern Chinese abacus except that the beads were not attached to bamboo bar rails. Though differently colored beads were used above and below the partition, it was not absolutely necessary. This model had existed until the early seventh century.

Historical continuity is obviously provided by Hsü Y'ao's work, by Ch'en Lan's Commentary and by full statutory recognition of Hsü Y'ao's work in the early Tang Dynasty and the then improvement of the abacus to exactly the modern pattern.

Perfection of the Chinese Abacus

From the initial period of the Tang Dynasty early in the seventh century, the Chu Pan had been constantly improved [30a] with vigor until the early part of the Sung Dynasty in the later part of the tenth century [30b]—by the perfection of the frame, by providing a hole through the middle of each bead, by

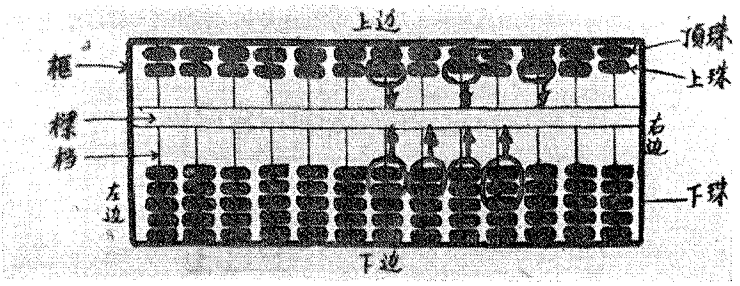

FIG. 4. The Abacus, Having Been in Use for Nearly 13 Centuries

setting a round bamboo rail at each digit, and by adding an additional bead both above and below the partition at each digit. The beads thus became attached within their boundaries for calculation operations, enabling the elimination of top and bottom recesses for storing beads. As in the earlier Chinese abacus with unattached beads, each bead above the partition denotes 5, and each below only "one". When 5 is reached, it is carried to one upper bead replacing five lower beads. When 10 is reached, it is carried to one lower bead on the left digit replacing two upper or one upper and five lower beads on the right digit. From the point of view of mere number representation, one of the two upper beads and one of the five lower beads appear redundant. It is, however, necessary in the advanced techniques of multiplication and division operations. Thus, it forms a bi-quinary device with digital increases from below and digital advances toward the left just as the writing of Chinese characters by adding toward the bottom and advancing to the next vertical line toward the left. This is the Chinese abacus which has been continuously in use for nearly 1,300 years without further improvement since the tenth century until very recently, and which is still in general use. Figure 4 shows this abacus with appropriate beads to be moved to represent the number 72,635 to take the operating positions adjacent to the partition.

At the time of the Sung Dynasty (960–1280 A.D.), an era of marked advancement of learning, successive generations of applied mathematicians [30b] made the use of the Chinese abacus widespread not only in China but also in the entire Orient.

In books written during the Yuan Dynasty (1280–1368 A.D.) and during the Ming Dynasty (1368–1644 A.D.), such as *General Rules of Mathematics* by Kê Shang-Chien [10, 30b], *Complete Commentary on Classified Calculation Methods of the Nine Books on Mathematics* by Wu Ching [11], and *Calculation Methods* by Cheng Ta-Wei [12, 30b] descriptions are given of the then Chinese abacus, as it still is today.

Development of Calculation Methods

The Chinese abacus is a most remarkable computer because, on the one hand, of its extreme simplicity, and, on the other, because of its most rapid methods

of calculation. These methods had their genesis in manipulating computation with calculating rods beginning more than 3,000 years ago, and they have been painstakingly developed and perfected ever since. As the Sung Dynasty came to pass, about 1,000 years ago, applied mathematicians had advanced the abacus mathematics to operating criteria verses. These verses yield partial arithmetical results during the calculation process. They represent mathematics of numbers and are in no way analogous to "programming" in the sense that it is needed in modern high-speed computers. All computing operations [24, 27, 30], whether addition, subtraction, multiplication, division; extracting square root, extracting cube root; area or volume evaluation; decimals, fractions, and ratios; interest, annuity, or sinking fund; trigonometric functions; or proof-checking routines, have been exactly expressed in concise criteria verses in terms of standard nomenclature especially developed for these methods of calculations. Having known a problem, the experienced computer, on glancing at the beads to be operated upon, will execute almost instantly the applicable criteria verses by actuating several relevant beads simultaneously with his fingers. The correct answer appears as soon as the operation of beads is finalized. Thus, the Chinese abacus provides a most efficient calculating device for any single numerical problem without repetitive runs.

Both the electronic digital computer and the Chinese abacus are mainly for arithmetic operations. Any computation that can be carried out on a digital computer can also be performed with the Chinese abacus. For instance, in matrix inversion for solving a large number of simultaneous equations, the new coefficients can be easily computed with the Chinese abacus at a speed as fast as one can write them algebraically.

Japanese Modification

Beginning with seventh century, there were at times as many as 2,000 Japanese students studying in the then Chinese capital at Chang-An, now called Siking. Knowledge of the Chinese abacus was thus soon extended to Japan and it came into general use in that country. Later, the Japanese dispensed with one of the upper beads, and in some cases also one of the lower beads [22, 30]. In ordinary arithmetical operations this is permissible, but in certain advanced operating techniques this limits the usefulness of the Chinese abacus. Another modification made by the Japanese is the addition of digit division points on the horizontal partition between every three digits.

Latest Improvements

In 1935 renewed improvement effort was made by computers in China [30]. King Chian-Ching advanced the King's new abacus. Lin Li-Chuan put out the Chiun-Heng abacus. The chief improvements consisted of the incorporation of fixed-point signal and the extension from the horizontal direction to the vertical direction. These improved versions of the Chinese abacus were in their infancy

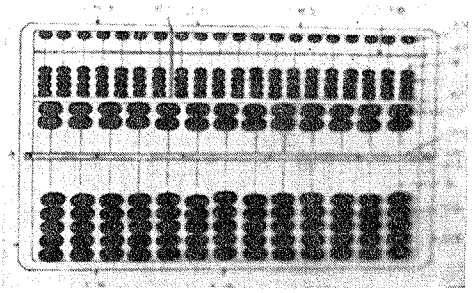

FIG. 5. Latest Compound Abacus

at the time of the Sino-Japanese War and World War II and hence have not yet come into general use.

In 1956, Li Kai-Cheng [29, 30] made the latest improvement by compounding two auxiliary abacuses on top of the main abacus, incorporating fixed-digit division points in both auxiliary abacuses, adding movable fixed-point and movable digit division points in the main abacus, thereby much enhancing the operating convenience and usefulness of the Chinese abacus. Figure 5 depicts this improved compound abacus.

In this Li's new compound abacus the calculation of the summation of a number of terms each involving multiplication and division operations can be carried out by using the main abacus as the arithmetical unit and the auxiliary abacuses for the storage of successive intermediate and final results.

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