Hexadecimals on the Soroban

Used in mathematics and computer sciences, hexadecimal or "hex" is a base 16 numerical system. It's main purpose is to represent the binary code in a way that makes it easier for most of us to read. In essence it acts as a kind of shorthand. In hexadecimal, the numbers 0-9 are represented at face value then; A=ten, B=eleven, C=twelve, D=thirteen, E=fourteen and F=fifteen. For example take the decimal numeral 61. It has a binary representation of 00111101 but is more simply expressed as 3D in hexadecimal (3 = 0011, D = 1101)

0 hex = 0 dec	0	0	0	0
1 hex = 1 dec	0	0	0	1
2 hex = 2 dec	0	0	1	0
3 hex = 3 dec	0	0	1	1
4 hex = 4 dec	0	1	0	0
5 hex = 5 dec	0	1	0	1
6 hex = 6 dec	0	1	1	0
7 hex = 7 dec	0	1	1	1
8 hex = 8 dec	1	0	0	0
9 hex = 9 dec	1	0	0	1
A hex = 10 dec	1	0	1	0
B hex = 11 dec	1	0	1	1
C hex = 12 dec	1	1	0	0
D hex = 13 dec	1	1	0	1
E hex = 14 dec	1	1	1	0
F hex = 15 dec	1	1	1	1

Conversion Chart

Binary Codes

A "binary code" is any system that uses only two states (0 or 1, on or off, true or false, etc.) The base 2 or binary number system is one example of a binary code.

How it works for numbers

Take the example 201, which can also be expressed as: 128 + 64 + 8 + 1

```
128 = 10000000
 64 =  1000000
  8 =     1000
  1 =        1
--------------
201 = 11001001
```

So 201 has a binary representation of 11001001. The problem is the binary number is not easily understandable to many of us. Converting 201 to hex makes the number more accessible. ***(see Example:1)***

Convert Base 10 to Hexadecimal

Basically it works like this:

- Divide the base 10 number by 16.
- Remainders on the right form part of the hexadecimal answer.
- If the quotient on the left is 16 or greater divide again to make it less than 16.

This is an advanced technique. It's important to have a good understanding of Takashi Kojima's methods for solving problems of *division* and in particular how to place *quotient answers*.

Example 1: Convert 201 to its hexadecimal and binary equivalents

Step 1: Set 201 and 16 onto the soroban. (Fig.1)

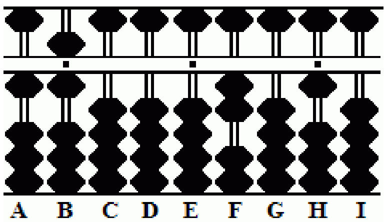

Fig.1

```
Step 1
A B C D E F G H I
_________________
  .     .     .
_________________
1 6 0 0 0 2 0 1 0
```

Step 2 and the answer: Divide 201 by 16. What remains on the soroban is the quotient 12 & the remainder 09. When converted to hex 12 09 become C9 with a binary representation of 11001001 (C = 1100, 9 = 1001). (Fig.2)

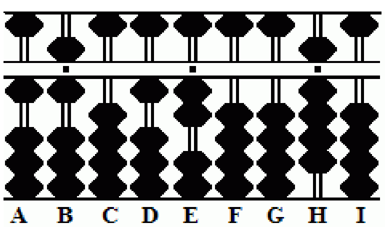

Fig.2

```
Step 2
A B C D E F G H I
_________________
  .     .     .
_________________
1 6 0 0 0 2 0 1 0
     (1)
        - 1 6
_________________
1 6 0 1 0 0 4 1 0
       (2)
          - 3 2
_________________
1 6 0 1 2 0 0 9 0
```

Example 2: Convert 2989 to its hexadecimal and binary equivalents (Fig.3)

Step1 : Set 2989 and 16 onto the soroban. (Fig.3)

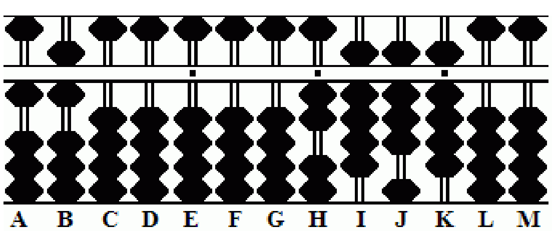

Fig.3

```
Step 1
A B C D E F G H I J K L M
_________________________
  .     .     .     .
_________________________
1 6 0 0 0 0 0 2 9 8 9 0 0
```

Step 2: Divide 2989 by 16 leaving the quotient answer 186 on rods FGH. Notice the remainder 13 on rods JK; it forms the last part of the hexadecimal answer. (Fig.4)

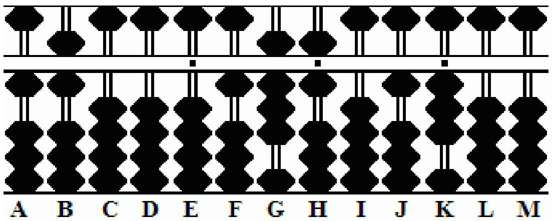

Fig.4

```
Step 2
A B C D E F G H I J K L M
-------------------------
  .     .     .     .
-------------------------
1 6 0 0 0 0 0 2 9 8 9 0 0
         (1)
            - 1 6
-------------------------
1 6 0 0 0 1 0 1 3 8 9 0 0
           (8)
            - 1 2 8
-------------------------
1 6 0 0 0 1 8 0 1 0 9 0 0
             (6)
                - 9 6
-------------------------
1 6 0 0 0 1 8 6 0 1 3 0 0
```

Step 3 and the answer: Convert 186 to hex by dividing again. With this last step what remains on the soroban is the quotient 11 and the remainders 10 & 13. When expressed in hex 11 10 13 becomes BAD with a binary representation of 101110101101 (B = 1011, A = 1010, D = 1101). (Fig.5)

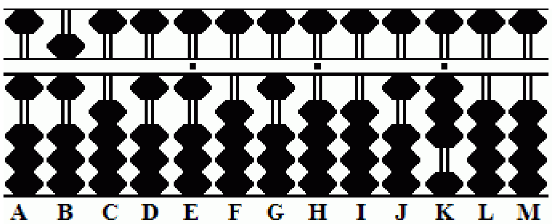

Fig.5

```
Step 3
A B C D E F G H I J K L M
-------------------------
  .     .     .     .
-------------------------
1 6 0 0 0 1 8 6 0 1 3 0 0
     (1)
        - 1 6
-------------------------
1 6 0 0 0 0 2 6 0 1 3 0 0
       (1)
          - 1 6
-------------------------
1 6 0 1 1 0 1 0 0 1 3 0 0
```

Convert Hexadecimal to Base 10

The technique for converting hexadecimal to base 10 is very much as the one above but in reverse. In other words instead of dividing we multiply in the reverse order.

- Set each hex number onto the soroban making sure to use unit rods for unit numbers.
- Work left to right starting with the left-hand most group.
- Multiply the unit number first, then the tens where applicable.
- Move to the next group and continue in the same fashion.

All this is best explained by example. It's really quite simple to do. Here's a simple example first.

This is an advanced technique. It's important to have a good understanding of Takashi Kojima's methods for solving problems of *multiplication* using the soroban.

Example 1: Convert the hex D0A to Base 10

Step 1: Hexadecimal D0A translates to 13 00 10. Set each group of numbers in such a way that each unit number falls on a unit rod. In this case 13 on rods GH, 00 on JK & 10 on MN. Set the multiplier 16 on rods AB.

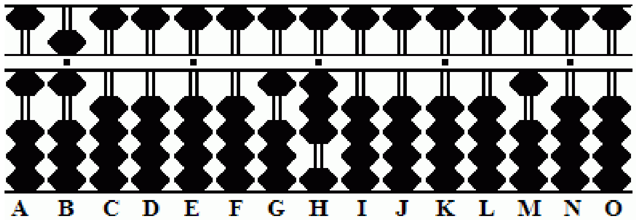

Fig.6

```
Step 1
A B C D E F G H I J K L M N O
  .     .     .     .     .
1 6 0 0 0 0 1 3 0 0 0 0 1 0 0
```

Step 2: Starting with the left most group multiply 3 on H by 16. Add the product 48 to rods JK. Clear 3 from H.

2a: Multiply 1 on rod G by 16. Add the product 16 to rods IJ. Clear 1 from rod G. (Fig.7)

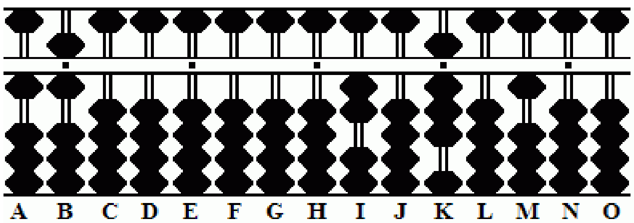

Fig.7

```
Steps 2 & 2a
A B C D E F G H I J K L M N O
  .     .     .     .     .
1 6 0 0 0 0 1 3 0 0 0 0 1 0 0
                + 4 8
clear       (-3)
1 6 0 0 0 0 1 0 0 4 8 0 0 0 0
              + 1 6
clear     (-1)
1 6 0 0 0 0 0 0 2 0 8 0 1 0 0
```

Step 3: Now move right to the next group and multiply 8 on K by 16. Add the product 128 to rods LMN. Clear 8 from K.

3a and the answer: Multiply 2 on I by 16. Add the product 32 to rods HI. Clear 2 from I and that completes the problem. Hexadecimal D0A converts to base its 10 number 3338. (Fig.8)

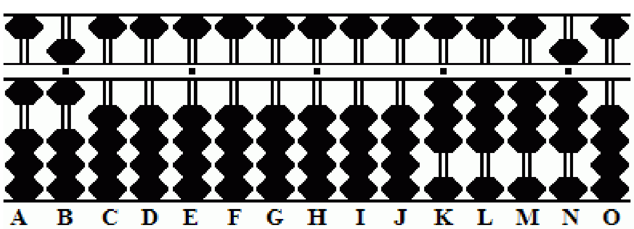

Fig.8

```
Steps 3 & 3a
A B C D E F G H I J K L M N O
  .     .     .     .     .
1 6 0 0 0 0 0 0 2 0 8 0 1 0 0
                    + 1 2 8
clear             (-8)
1 6 0 0 0 2 0 0 2 0 0 1 3 8 0
                  + 3 2
clear   (-2)
1 6 0 0 0 0 0 0 0 0 3 3 3 8 0
```

Example 2: Convert the hex B4E8 to Base 10

Step 1: Hexadecimal B4E8 translates to 11 04 14 08. As before set each group so that each unit number falls on a unit rod; in this case 11 on DE, 04 on GH, 14 on JK & 08 on MN. (Fig.9)

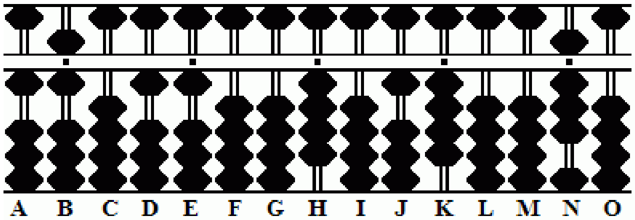

Fig.9

```
Step 1
A B C D E F G H I J K L M N O
  .     .     .     .     .
1 6 0 1 1 0 0 4 0 1 4 0 0 8 0
```

Step 2: Starting with the left most multiply 1 on E by16. Add the product 16 to rods GH. Clear 1 from rod E.

2a: Multiply 1 on D by 16. Add the product 16 to rods FG. Clear 1 from rod D. (Fig.10)

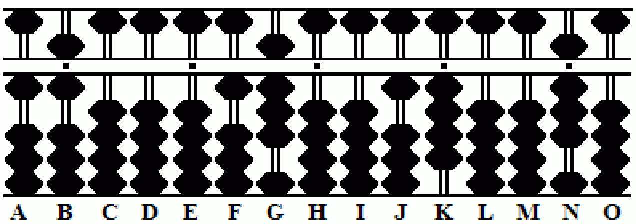

Fig.10

```
Steps 3 & 3a
A B C D E F G H I J K L M N O
  .     .     .     .     .
1 6 0 0 0 1 8 0 0 1 4 0 0 8 0
            + 1 2 8
clear     (-8)
1 6 0 0 0 1 0 1 2 9 4 0 0 8 0
            + 1 6
clear   (-1)
1 6 0 0 0 0 0 2 8 9 4 0 0 8 0
0
```

Step 3: Now move right to the next group to and multiply 8 on Gt by 16 Add the product 128 to rods HIJ. Clear 8 from G.

3a: Multiply 1 on rod F by 16. Add the product 16 to rods HI. Clear 1 from rod F. (Fig.11)

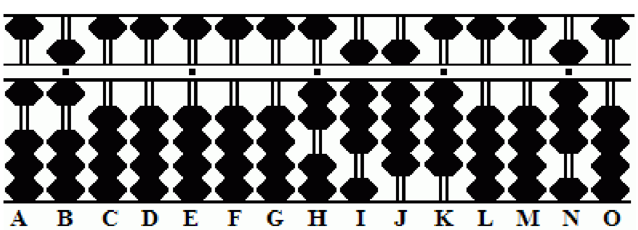

Fig.11

```
Steps 3 & 3a
A B C D E F G H I J K L M N O
  .     .     .     .     .
1 6 0 0 0 1 8 0 0 1 4 0 0 8 0
            + 1 2 8
clear     (-8)
1 6 0 0 0 1 0 1 2 9 4 0 0 8 0
            + 1 6
clear   (-1)
1 6 0 0 0 0 0 2 8 9 4 0 0 8 0
```

Step 4: Move to the next group and multiply 4 on rod K by 16. Add the product 64 to rods MN. Clear 4 from K.

4a: Multiply 9 on J by 16 and add the product 144 to KLM. Clear 9 from J.

4b: Multiply 8 on rod I by 16, add the product 128 to JKL. Clear 8 from I

4c and the answer: Multiply 2 on on H by16 and add the product 32 to rods IJ. Clear 2 from H and that completes the problem. Hexadecimal B4E8 converts to its base 10 number 46312.

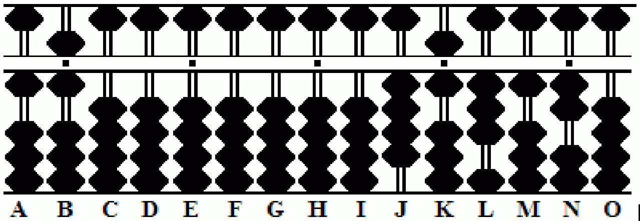

Fig.12

```
Steps 4 4a 4b & 4c
A B C D E F G H I J K L M N O
-----------------------------
  .     .     .     .     .
-----------------------------
1 6 0 0 0 0 0 2 8 9 4 0 0 8 0
                       +6 4
clear             (-4)
-----------------------------
1 6 0 0 0 0 0 2 8 9 0 0 7 2 0
                  + 1 4 4
clear           (-9)
-----------------------------
1 6 0 0 0 0 0 2 8 0 1 5 1 2 0
                + 1 2 8
clear         (-8)
-----------------------------
1 6 0 0 0 0 0 2 0 1 4 3 1 2 0
                + 3 2
clear       (-2)
-----------------------------
1 6 0 0 0 0 0 0 0 4 6 3 1 2 0
```

A word of thanks

I hadn't really thought much about converting back and forth between base 10 and hex before last September when one day Shane Baggs shared his techniques in a *post* to members of the Yahoo *Soroban/Abacus* group. Since then with a little study and working closely with Shane's techniques, I've learned more about converting hex, base 10 and binary than I ever thought possible. I love new stuff.

Sincere thanks, Totton Heffelfinger

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